M2 JUNE 13 uu

1. A particle P of mass 2 kg is moving with velocity $(\mathbf{i} - 4\mathbf{j})$ m s^{-1} when it receives an impulse of $(3\mathbf{i} + 6\mathbf{j})$ N s.

Find the speed of P immediately after the impulse is applied.

(5)

Momentum = mass x vel

Initial mom $= 2\begin{pmatrix}1\\-4\end{pmatrix} = \begin{pmatrix}2\\-8\end{pmatrix}$

final Mom = Initial mom + Impulse $= \begin{pmatrix}2\\-8\end{pmatrix} + \begin{pmatrix}3\\6\end{pmatrix}$

final mom $= \begin{pmatrix}5\\-2\end{pmatrix} = 2\begin{pmatrix}x\\y\end{pmatrix}$ $\therefore$ final speed $\begin{pmatrix}2.5\\-1\end{pmatrix}$

$\sqrt{2.5^2+1^2} = \underline{2.69 \text{ ms}^{-1}}$ (3sf)

2. A particle P of mass 3 kg moves from point A to point B up a line of greatest slope of a fixed rough plane. The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4

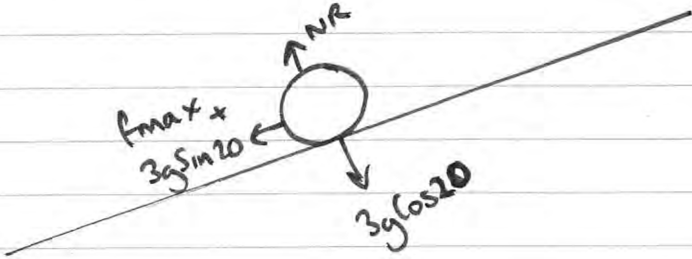
Given that $AB = 15$ m and that the speed of P at A is 20 m s^{-1}, find

(a) the work done against friction as P moves from A to B,

(3)

(b) the speed of P at B.

(4)



$\mu = 0.4$

$f_{max} = \mu NR = 0.4(3g\cos 20) = 11.05078522\ldots$

$\therefore$ wd against friction $\overrightarrow{AB}$ = $f_{max} \times 15$ = 166J (3sf)

b) KE_A - wd against friction $= KE_B + PE_B$

$\frac{1}{2}(3)20^2 - 165.761\ldots = \frac{1}{2}(3)v^2 + 3g(15\sin 20)$

$\Rightarrow \quad \frac{3}{2}v^2 = 283.407\ldots \qquad \Rightarrow \underline{v = 13.7 \text{ ms}^{-1}}$

3. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geqslant 0$$

Find

(a) the times when P is instantaneously at rest,
(3)

(b) the greatest speed of P in the interval $0 \leqslant t \leqslant 4$
(5)

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$
(5)

a) $2t^2 - 14t + 20 = 0$

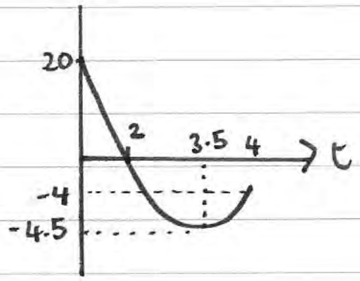
$(2t - 4)(t - 5) = 0$

$t = 2 \quad t = 5$

b) $a = \frac{dv}{dt} = 4t - 14 \Rightarrow$ when $a = 0 \quad t = \frac{7}{2}$

$v = 2\left(\frac{7}{2}\right)^2 - 14\left(\frac{7}{2}\right) + 20 = -4.5$

$\therefore$ greatest speed is when $t = 0$, $V = 20$ Max Speed $= 20$.



c) $S = \int v\,dt = \frac{2}{3}t^3 - 7t^2 + 20t + C \qquad S = 0, t = 0 \Rightarrow C = 0$

$\int_0^2 v\,dt = \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_0^2 = \frac{52}{3}$

$\int_2^4 v\,dt = \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_2^4 = \frac{32}{3} - \frac{52}{3} = \frac{-20}{3}$

$\therefore$ total distance $= \frac{52}{3} + \frac{20}{3} = \frac{72}{3} = 24$ m

4.

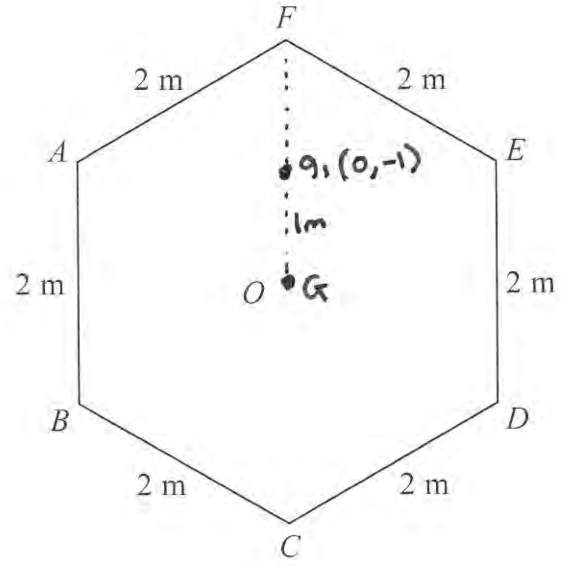


Figure 1

The uniform lamina *ABCDEF* is a regular hexagon with centre *O* and sides of length 2 m, as shown in Figure 1.

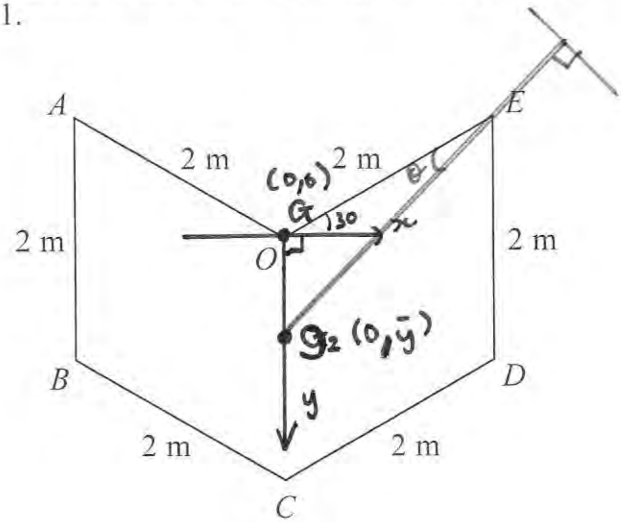


Figure 2

The triangles *OAF* and *OEF* are removed to form the uniform lamina *OABCDE*, shown in Figure 2.

(a) Find the distance of the centre of mass of *OABCDE* from *O*.

(5)

The lamina *OABCDE* is freely suspended from *E* and hangs in equilibrium.

(b) Find the size of the angle between *EO* and the downward vertical.

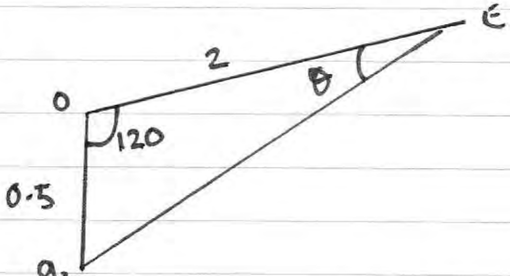
(6)

6 equalateral triangles each with mass m.

$\therefore$ x: $4mg \times \bar{y} + 2mg \times -1 = 6mg \times 0$.

$\Rightarrow 4\cancel{mg}\,\bar{y} = 2\cancel{mg} \qquad \bar{y} = \frac{1}{2}$ m

b)



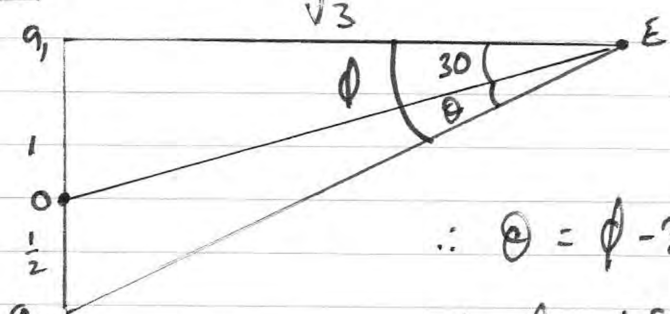
$g_2E^2 = 2^2 + 0.5^2 - 2^2(\frac{1}{2})\cos 120$

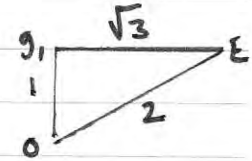
$\therefore g_2E = \sqrt{\frac{21}{4}}$

$$\frac{\sin\theta}{\frac{1}{2}} = \frac{\sin 120}{g_2E}$$

$\therefore \theta = \underline{10.9^\circ}$

alt





$\therefore \theta = \phi - 30$

$\tan\phi = \frac{1.5}{\sqrt{3}} = 40.893\ldots$

$\therefore \theta = \underline{10.9^\circ}$

5.

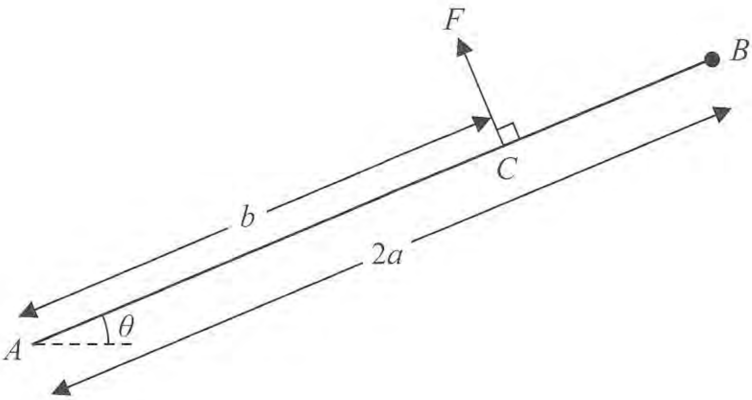


Figure 3

A uniform rod AB, of mass m and length $2a$, is freely hinged to a fixed point A. A particle of mass m is attached to the rod at B. The rod is held in equilibrium at an angle θ to the horizontal by a force of magnitude F acting at the point C on the rod, where $AC = b$, as shown in Figure 3. The force at C acts at right angles to AB and in the vertical plane containing AB.

(a) Show that $F = \dfrac{3amg\cos\theta}{b}$.

(4)

(b) Find, in terms of a, b, g, m and θ,

(i) the horizontal component of the force acting on the rod at A,

(ii) the vertical component of the force acting on the rod at A.

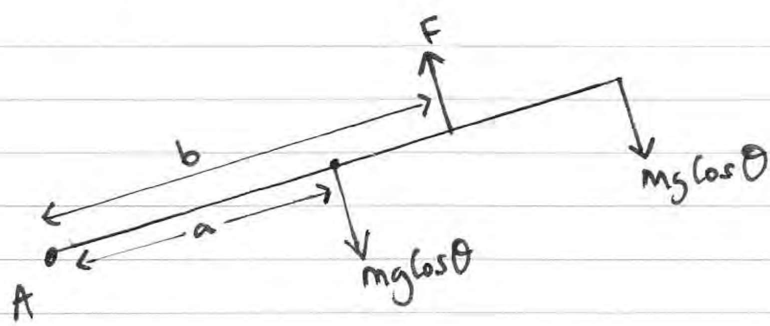
(5)

Given that the force acting on the rod at A acts along the rod,

(c) find the value of $\dfrac{a}{b}$.

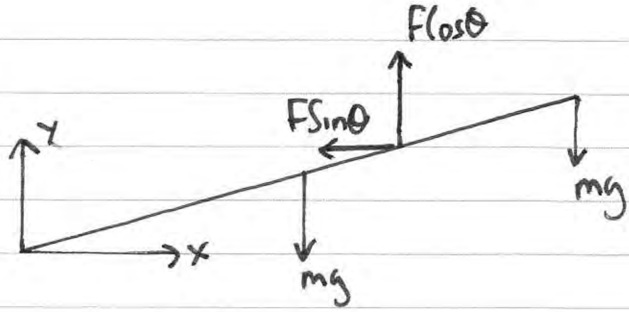
(4)

a)



$A \circlearrowright \quad mg\cos\theta \times a + mg\cos\theta \times 2a = F \times b$

$\therefore \quad F = \dfrac{3mg\cos\theta \times a}{b}.$



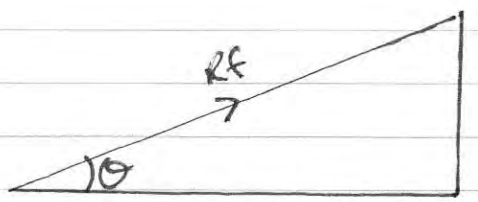
$R_f \uparrow = 0 \Rightarrow Y = 2mg - F\cos\theta$

$\overrightarrow{R_f} = 0 \Rightarrow X = F\sin\theta$

i) $X = \dfrac{3amg\sin\theta\cos\theta}{b}$

ii) $Y = 2mg - \dfrac{3amg\cos^2\theta}{b}$

c)



$2mg - \dfrac{3amg\cos^2\theta}{b}$

$\dfrac{3amg\sin\theta\cos\theta}{b}$

$\therefore \tan\theta = \dfrac{\left(\dfrac{2mgb - 3amg\cos^2\theta}{\cancel{b}}\right)}{\dfrac{3amg\sin\theta\cos\theta}{\cancel{b}}}$

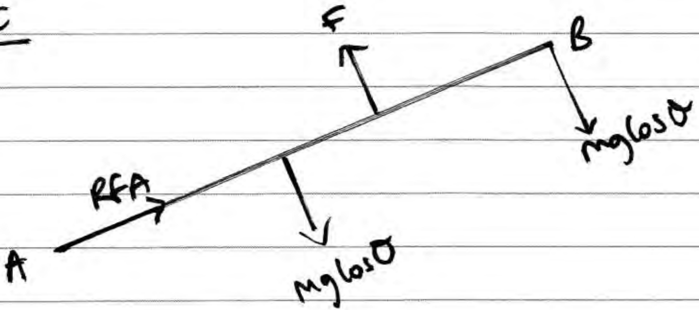
$\Rightarrow \dfrac{\sin\theta}{\cancel{\cos\theta}} \times 3amg\sin\theta\cancel{\cos\theta} = 2mgb - 3amg\cos^2\theta$

$3a\cancel{mg}\sin^2\theta = 2\cancel{mg}b - 3a\cancel{mg}\cos^2\theta$

$3a\sin^2\theta = 2b - 3a\cos^2\theta$

$\Rightarrow 2b = 3a(\underbrace{\sin^2\theta + \cos^2\theta}) \Rightarrow \dfrac{a}{b} = \dfrac{2}{3}$

c) alt



$$B \circlearrowleft \quad F \times (2a-b) = mg\cos\theta \times a$$

$$(2a-b)\,3\,\frac{\cancel{a}\cancel{m}\cancel{g}\cancel{\cos\theta}}{b} = \cancel{m}\cancel{g}\cancel{\cos\theta} \times \cancel{a}$$

$$(2a-b)\,3 = b \Rightarrow 6a = 4b \Rightarrow \frac{a}{b} = \frac{2}{3}$$

6.

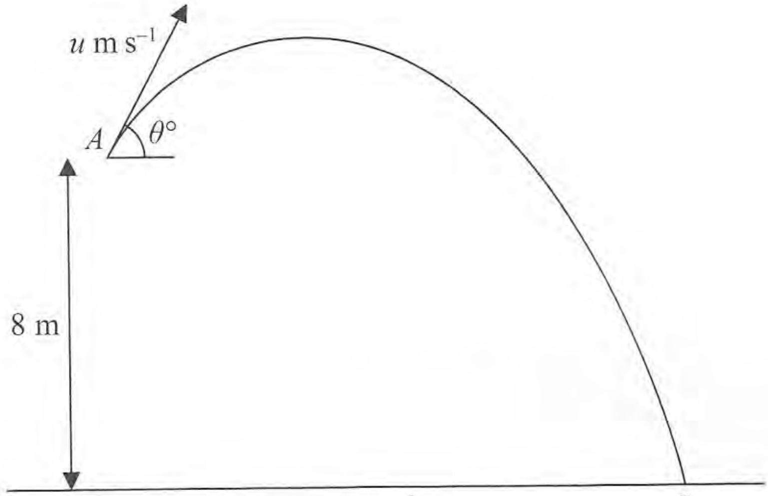


Figure 4

A ball is projected from a point A which is 8 m above horizontal ground as shown in Figure 4. The ball is projected with speed u m s^{-1} at an angle $\theta°$ above the horizontal. The ball moves freely under gravity and hits the ground at the point B. The speed of the ball immediately before it hits the ground is $2u$ m s^{-1}.

(a) By considering energy, find the value of u.

(5)

The time taken for the ball to move from A to B is 2 seconds. Find

(b) the value of θ,

(4)

(c) the minimum speed of the ball on its path from A to B.

(2)

a) $KE_A + PE_A = KE_B$

$\Rightarrow \frac{1}{2}\cancel{m}u^2 + \cancel{m}g \times 8 = \frac{1}{2}\cancel{m}(2u)^2 \Rightarrow \frac{1}{2}u^2 + 8g = 2u^2$

$\Rightarrow \frac{3}{2}u^2 = 8g \Rightarrow u = \sqrt{\frac{16g}{3}} = 7.23\text{ms}^{-1}$ (3sf)

b) $\overrightarrow{H}$ vel $= u\cos\theta$ $\quad$ $V\uparrow$ $\quad$ $s = -8$

dist $= x$ $\quad$ $u = u\sin\theta$

time $= 2$ $\quad$ $v =$

$a = -9.8$

$t = 2$

$s = ut + \frac{1}{2}at^2 \Rightarrow -8 = 2u\sin\theta - 4.9 \times 2^2$

$\Rightarrow \sin\theta = \frac{11.6}{2u} \Rightarrow \theta = \underline{53.3°}$

c) horizontal speed is constant $\therefore$ min speed is when vertical speed is zero

$\therefore$ min speed $u\cos\theta = \underline{4.32\ ms^{-1}}$

c) horizontal speed is constant $\therefore$ min speed is when vertical speed is zero

$\therefore$ min speed $u\cos\theta = \underline{4.32\ ms^{-1}}$

7. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q and R have masses $2m$, $3m$ and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(a) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1+e)u$.

(6)

After the collision between P and Q there is a direct collision between Q and R.

Given that $e = \frac{3}{4}$, find

(b) (i) the speed of Q after this collision,

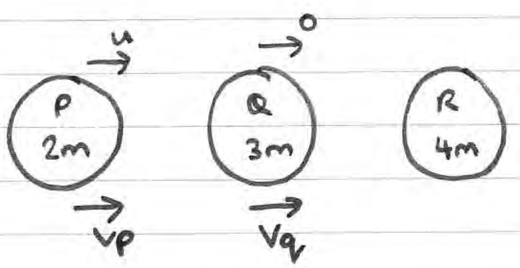
(ii) the speed of R after this collision.

(6)

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(c) Find V in terms of u.

(3)



sep $= V_q - V_p$

approach $= u$

$\therefore \quad e = \dfrac{V_q - V_p}{u}$

$V_q \geq V_p$

$V_q - V_p = eu$

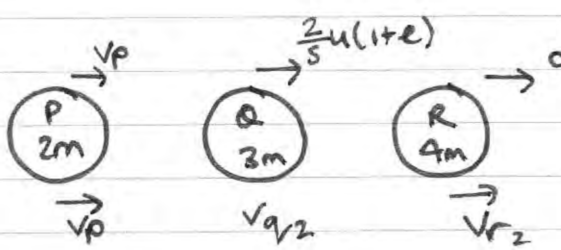
$V_p = V_q - eu$

CLM $\quad 2\cancel{m}u = 2\cancel{m}V_p + 3\cancel{m}V_q$

$2u = 2V_q - 2eu + 3V_q$

$\Rightarrow 5V_q = 2u(1+e) \qquad \therefore V_q = \dfrac{2u}{5}(1+e)$ #

b)



$\frac{2}{5}u\left(\frac{7}{4}\right) = \frac{7}{10}u$

$e = \dfrac{3}{4} = \dfrac{V_{r_2} - V_{q_2}}{\frac{7}{10}u}$

CLM $\quad 3m\left(\frac{7}{10}u\right) = 3\cancel{m}V_{q_2} + 4mV_{r_2}$

$21\cancel{m}u = 30\cancel{m}V_{q_2} + 40\cancel{m}V_{r_2}$

$21u = 40V_{r_2} - 40V_{q_2}$

$21u = 30V_{q_2} + 40V_{r_2}$ and $21u = 40V_{r_2} - 40V_{q_2}$

$\Rightarrow 40V_{r_2} = 21u + 40V_{q_2}$

$\cancel{21u} = 30V_{q_2} + \cancel{21u} + 40V_{q_2}$

$\therefore\ 30V_{q_2} = 40V_{q_2} \Rightarrow V_{q_2} = 0$

$\Rightarrow 21u = 40V_{r_2} \Rightarrow V_{r_2} = \frac{21}{40}u$

c) $V_p = V_q - eu = \frac{7}{10}u - \frac{3}{4}u = -\frac{1}{20}u$

P: $\xleftarrow{\quad}$ $\frac{2}{40}u$ $\qquad$ R: $\xrightarrow{\quad}$ $\frac{21}{40}u$

$\therefore\ V = \frac{23}{40}u.$