

M2 May 2012

1. [In this question \mathbf{i} and \mathbf{j} are perpendicular unit vectors in a horizontal plane.]

A particle P moves in such a way that its velocity \mathbf{v} in m s^{-1} at time t seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}$$

(a) Find the magnitude of the acceleration of P when $t = 1$

(5)

Given that, when $t = 0$, the position vector of P is \mathbf{i} metres,

(b) find the position vector of P when $t = 3$

(5)

$$\text{a) } \mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + (4 - 2t)\mathbf{j}; \quad t=1 \quad \mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{6^2 + 2^2} = \underline{6.32 \text{ ms}^{-2}} \quad (3 \text{ sf})$$

$$\text{b) } \mathbf{s} = \int \mathbf{v} dt = (t^3 - t + C_1)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + C_2)\mathbf{j};$$

$$t=0 \quad \mathbf{i}=1 \quad \mathbf{j}=0 \quad \begin{aligned} 0^3 - 0 + C_1 &= 1 \quad \therefore C_1 = 1 \\ 2 \times 0^2 - \frac{1}{3}0^3 + C_2 &= 0 \quad \therefore C_2 = 0 \end{aligned}$$

$$\mathbf{s} = (t^3 - t + 1)\mathbf{i} + (2t^2 - \frac{t^3}{3})\mathbf{j};$$

$$t=3 \quad \mathbf{s} = 25\mathbf{i} + 9\mathbf{j}$$

2. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P . The coefficient of restitution between P and Q is e .

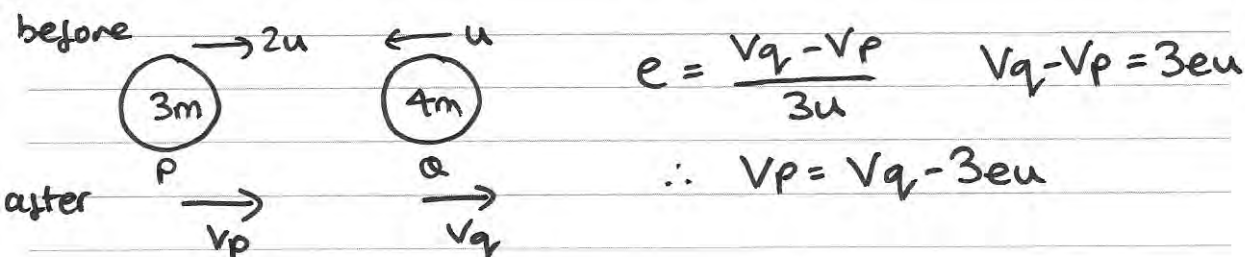
(a) Find the speed of Q immediately after the collision.

(6)

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e .

(5)



CLM $6mu - 4mu = 3mv_p + 4mv_q$

$$2u = 3v_q - 9eu + 4v_q \Rightarrow 7v_q = 2u + 9eu$$

$$\therefore v_q = \frac{1}{7}u(2 + 9e)$$

b) $v_p < 0 \Rightarrow \frac{2}{7}u + \frac{9}{7}eu - 3eu < 0$

$$\Rightarrow \frac{2}{7}u < \frac{12}{7}eu \Rightarrow 2 < 12e \Rightarrow e > \frac{1}{6}$$

$$\left(\therefore \frac{1}{6} < e \leq 1 \right)$$

3.

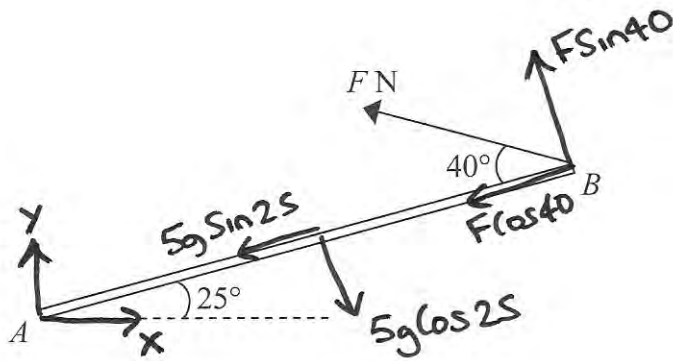


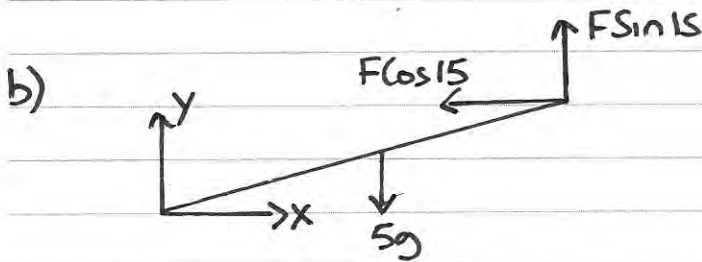
Figure 1

A uniform rod AB , of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B . The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

- (a) Find the value of F . (4)
- (b) Find the magnitude and direction of the vertical component of the force acting on the rod at A . (4)

$$a) \quad \overset{\curvearrowright}{A} \quad 5g \cos 25 \times 2 = F \sin 40 \times 4$$

$$\therefore F = \frac{10g \cos 25}{4 \sin 40} = 34.5 \text{ N (3sf)}$$



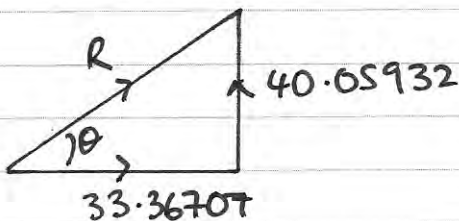
$$R \uparrow = 0$$

$$Y = 5g - F \sin 15$$

$$Y = 40.05932044$$

$$\vec{R} \uparrow = 0$$

$$X = F \cos 15 = 33.36707$$



$$R = \sqrt{33.36707^2 + 40.05932^2}$$

$$R = 52.1 \text{ N (3sf)}$$

$$\theta = \tan^{-1} \left(\frac{40.05932}{33.36707} \right)$$

$$\theta = 50.2^\circ \text{ above horizontal}$$

4.

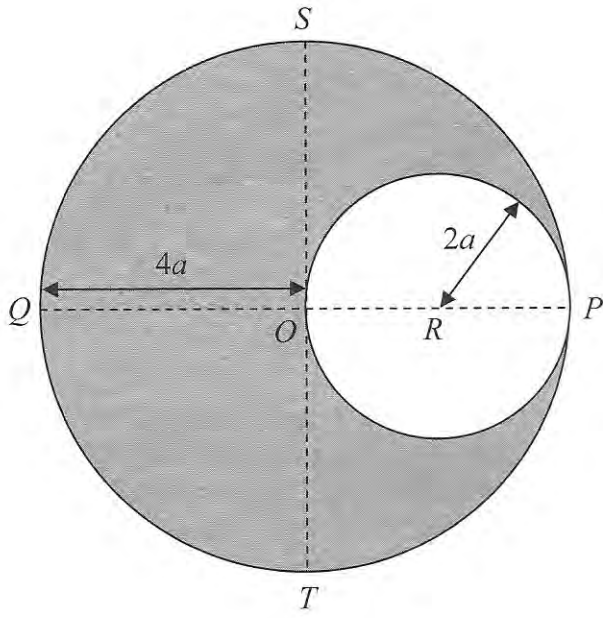


Figure 2

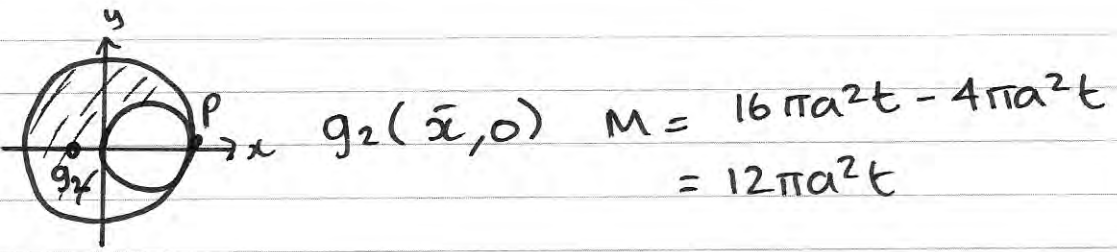
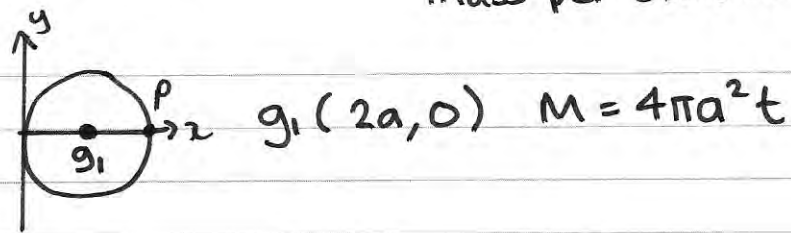
A uniform circular disc has centre O and radius $4a$. The lines PQ and ST are perpendicular diameters of the disc. A circular hole of radius $2a$ is made in the disc, with the centre of the hole at the point R on OP where $OR = 2a$, to form the lamina L , shown shaded in Figure 2.

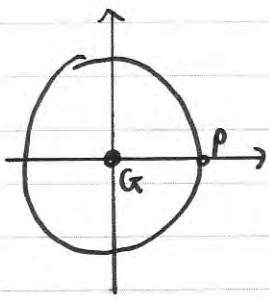
(a) Show that the distance of the centre of mass of L from P is $\frac{14a}{3}$. (4)

The mass of L is m and a particle of mass km is now fixed to L at the point P . The system is now suspended from the point S and hangs freely in equilibrium. The diameter ST makes an angle α with the downward vertical through S , where $\tan \alpha = \frac{5}{6}$.

(b) Find the value of k .

mass per unit area = t (5)





$$G(0,0) \quad M = 16\pi a^2 t$$

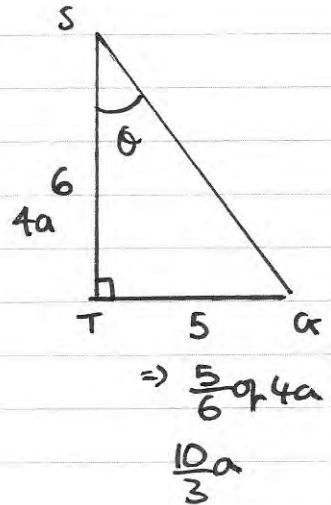
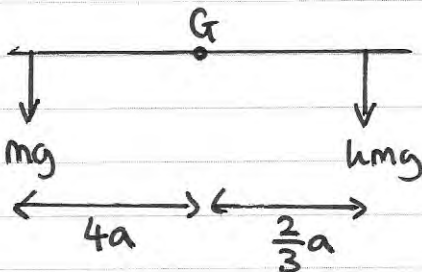
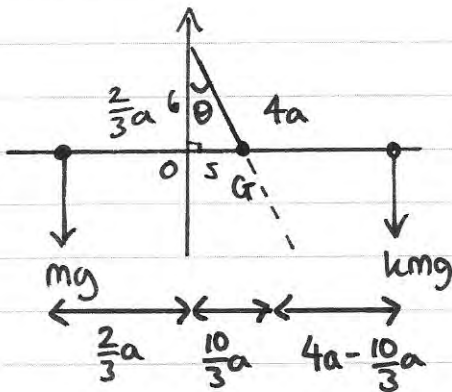
$$4\pi a^2 t g \times 2a + 12\pi a^2 t g \times \bar{x} = 16\pi a^2 t g \times 0$$

$$8a^3 + 12a^2 \bar{x} = 0 \Rightarrow 8a = -12\bar{x}$$

$$\therefore \bar{x} = -\frac{2}{3}a$$

$$x \text{ coordinate of } G_2 \text{ from } P = 4a + \frac{2}{3}a = \frac{14}{3}a \quad \#$$

b)



$$\overset{G}{\curvearrowright} \quad mg \times 4a = kmg \times \frac{2}{3}a$$

$$4 = \frac{2}{3}k$$

$$\therefore \underline{k = 6}$$

5.

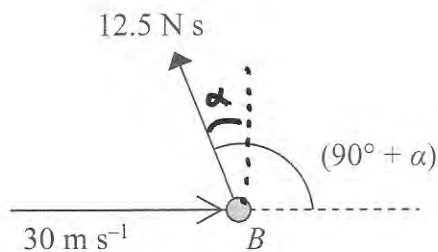


Figure 3

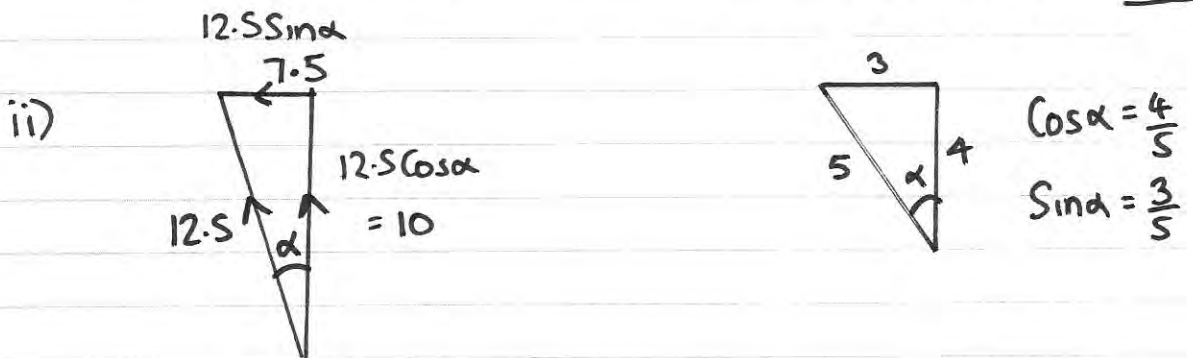
A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90^\circ + \alpha)$, where $\tan \alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

- (i) Find the speed of B immediately after the impulse is applied.
 (ii) Find the direction of motion of B immediately after the impulse is applied.

(6)

i) Momentum before = $0.25 \times 30 = 7.5 \text{ N s}$
 Impulse = change in momentum = 12.5

Momentum after = $0.25v = -5$
 $v = -20 \therefore \text{Speed} = \underline{20 \text{ m s}^{-1}}$



$$\text{Initial Mom} = 0.25 \begin{pmatrix} 30 \\ 0 \end{pmatrix} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$$

$$\text{Impulse} = \begin{pmatrix} -7.5 \\ 10 \end{pmatrix}$$

$$\text{final mom} = \begin{pmatrix} 0 \\ 10 \end{pmatrix} = 0.25v \therefore v = \begin{pmatrix} 0 \\ 40 \end{pmatrix}$$

change in
mom = Impulse

speed = 40 m s^{-1} due North

6. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{1}{14}$. The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW. The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer.

At a given instant, the car is moving at 10 m s^{-1} . Find

- (a) the acceleration of the car at this instant,

(5)

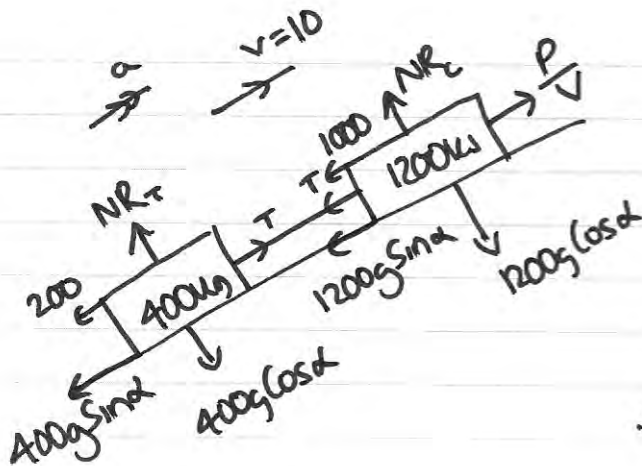
- (b) the tension in the towbar at this instant.

(4)

The towbar breaks when the car is moving at 12 m s^{-1} .

- (c) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest.

(5)



$$\vec{R}F = ma$$

$$\frac{60000}{10} - 1000 - 200$$

$$- 400g \times \frac{1}{14} - 1200g \times \frac{1}{14} = 1600a$$

$$\therefore 3680 = 1600a$$

$$a = \underline{2.3 \text{ ms}^{-2}}$$

b) trailer

$$\vec{R}F = ma \Rightarrow T - 200 - 400g \times \frac{1}{14} = 400 \times 2.3$$

$$T - 480 = 920$$

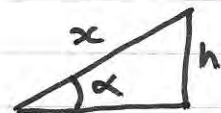
$$\therefore T = \underline{1400 \text{ N}}$$

c) Initial KE = final PE + Wd against Res

$$\frac{1}{2}(400) \times 12^2 = mg \left(\frac{1}{14}x \right) + 200 \times x$$

$$28800 = 280x + 200x = 480x$$

$$\therefore x = 60 \text{ m.}$$



$$\sin \alpha = \frac{1}{14} = \frac{h}{x}$$

$$h = \frac{1}{14}x$$

7.

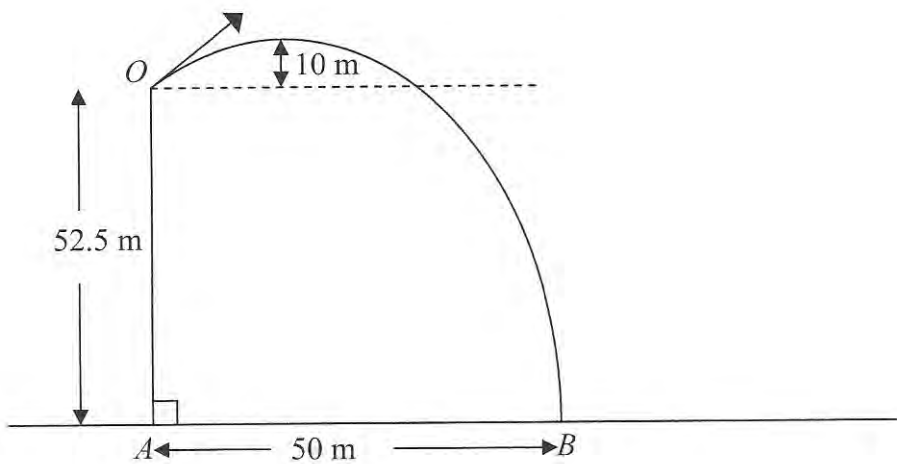


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA . The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B , where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

- (a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1} . (3)
- (b) Find the speed of projection. (9)
- (c) Find the time after projection when the stone is moving parallel to OB . (5)

$$\begin{aligned}
 \text{a) } u \uparrow & & v^2 &= u^2 + 2as \\
 s \uparrow &= 10 & 0 &= u^2 - 19.6 \times 10 \\
 a \uparrow &= -9.8 & \therefore u^2 &= 196 \\
 v \uparrow &= 0 & \therefore u &= \underline{14 \text{ m s}^{-1}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } u \uparrow &= 14 & s &= ut + \frac{1}{2}at^2 \\
 s \uparrow &= -52.5 & -52.5 &= 14t - 4.9t^2 \\
 a \uparrow &= -9.8 & 4.9t^2 - 140t - 525 &= 0 \\
 & & 7t^2 - 20t - 75 &= 0 \\
 & & (t-5)(7t+15) &= 0 \\
 & & \therefore t &= 5
 \end{aligned}$$

$$\vec{H} \quad \text{Dist} = \text{vel} \times \text{time}$$

$$50 = \text{vel} \times 5 \Rightarrow V_h = 10$$

$$V_h = 10, V_v = 14 \quad \text{speed} = \sqrt{14^2 + 10^2} = 17.2 \text{ms}^{-1}$$

(3sf)

$$OB = \begin{pmatrix} 50 \\ -52.5 \end{pmatrix}$$

$$\text{vel} = k \begin{pmatrix} 50 \\ -52.5 \end{pmatrix} = \begin{pmatrix} 50k \\ -52.5k \end{pmatrix}$$

$$V_h = 10 \therefore k = \frac{1}{5} \Rightarrow V_v = -10.5$$

$$u \uparrow = 14$$

$$a \uparrow = -9.8$$

$$v \uparrow = -10.5$$

$$v = u + at$$

$$-10.5 = 14 - 9.8t$$

$$-24.5 = -9.8t \quad \therefore t = \underline{\underline{2.5 \text{sec}}}$$