M2 JUNE 2010

1. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geqslant 0$, is $(3t + 5)\ \mathrm{m\,s^{-2}}$ in the positive x-direction. When $t = 0$, the velocity of P is $2\ \mathrm{m\,s^{-1}}$ in the positive x-direction. When $t = T$, the velocity of P is $6\ \mathrm{m\,s^{-1}}$ in the positive x-direction. Find the value of T.

(6)

$\text{acc} = 3t + 5$

$\Rightarrow \text{vel} = \int 3t + 5 \; dt = \frac{3t^2}{2} + 5t + C$

$V = 2, t = 0 \Rightarrow C = 2 \Rightarrow \text{vel} = \frac{3t^2}{2} + 5t + 2$

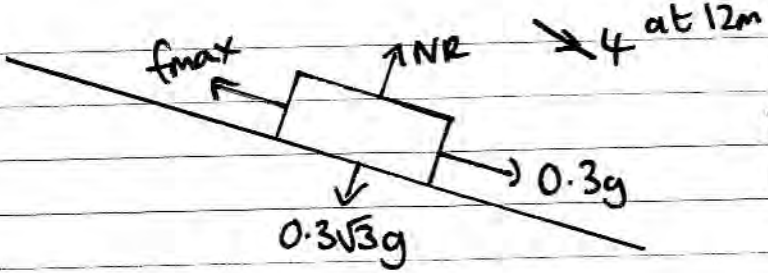
when $V = 6 \Rightarrow 6 = \frac{3}{2}t^2 + 5t + 2 \Rightarrow 3t^2 + 10t - 8 = 0$

$(3t-2)(t+4) = 0 \Rightarrow t = \frac{2}{3}$ sec

2. A particle P of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P has moved 12 m, its speed is 4 m s^{-1}. Given that friction is the only non-gravitational resistive force acting on P, find

(a) the work done against friction as the speed of P increases from 0 m s^{-1} to 4 m s^{-1}, **(4)**

(b) the coefficient of friction between the particle and the plane. **(4)**



$u=0 \quad v=4 \quad s=12$

$v^2=u^2+2as$

$16 = 2a(12) \quad a=\frac{2}{3}$

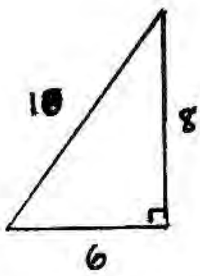
RF↘ $\quad 0.3g - f_{max} = 0.6 \times \frac{2}{3} \Rightarrow f_{max} = 0.3g - 0.4$

wd against friction $= (0.3g-0.4)\times 12 = \underline{30.5\,J}$ (3sf)

b) $f_{max} = \mu NR \quad 0.3g - 0.4 = \mu(0.3\sqrt{3}g)$

$$\mu = \frac{0.3g-0.4}{0.3\sqrt{3}g} = 0.499 \text{ (3sf)}$$

3.



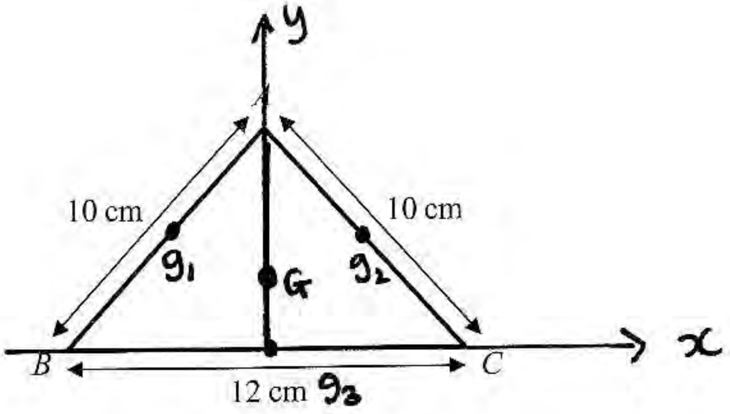


Figure 1

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

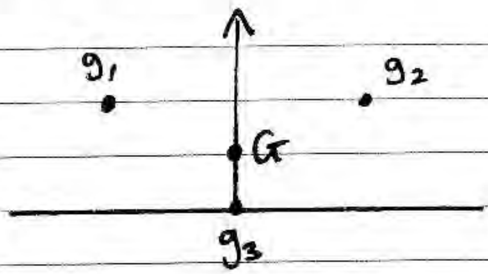
(a) Find the distance of the centre of mass of the frame from BC.

(5)

The frame has total mass M. A particle of mass M is attached to the frame at the mid-point of BC. The frame is then freely suspended from B and hangs in equilibrium.

(b) Find the size of the angle between BC and the vertical.

(4)



Mass ratio $10:10:12$

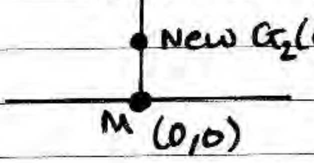
$\Rightarrow 5:5:6$

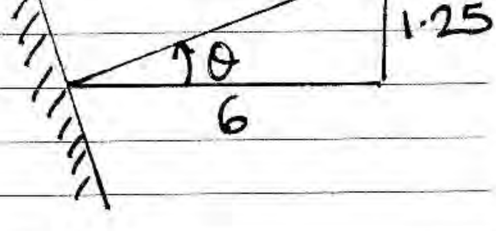
$g_1(-3,4)$ $g_2(3,4)$ $g_3(0,0)$

$G(0,\bar{y})$

$\curvearrowright x$ $\quad 5\times4+5\times4+6\times0 = 16\times\bar{y} \Rightarrow \bar{y} = 2.5\text{cm}$

b)





$\theta = \tan^{-1}\left(\frac{1.25}{6}\right) = 11.8^\circ$ (3sf)

4. A car of mass 750 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$. The resistance to motion of the car from non-gravitational forces has constant magnitude R newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of 20 m s^{-1}.

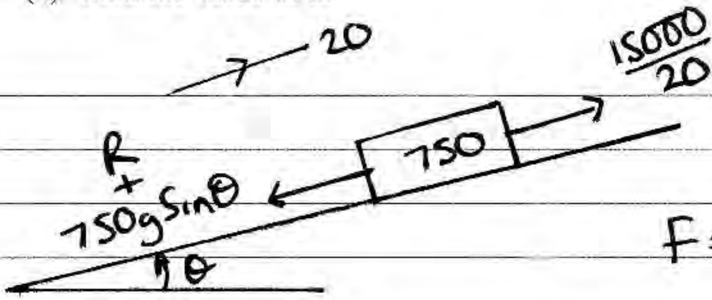
(a) Show that $R = 260$.

(4)

The power developed by the car's engine is now increased to 18 kW. The magnitude of the resistance to motion from non-gravitational forces remains at 260 N. At the instant when the car is moving up the road at 20 m s^{-1} the car's acceleration is a m s^{-2}.

(b) Find the value of a.

(4)



$\sin\theta = \frac{1}{15}$

$F = \frac{P}{V}$

$R_f \nearrow = 0 \Rightarrow 750 = 50g + R \Rightarrow R = 260\text{N}$

b) $P = 18\text{kW}$ $\quad \frac{18000}{20} - 260 - 50g = 750a$

$R_f = ma$

$a = \frac{1}{5}\text{ms}^{-2}$

5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity $(10\mathbf{i} + 24\mathbf{j})\ \mathrm{m\,s^{-1}}$ when it is struck by a bat. Immediately after the impact the ball is moving with velocity $20\mathbf{i}\ \mathrm{m\,s^{-1}}$.

Find

(a) the magnitude of the impulse of the bat on the ball,

(4)

(b) the size of the angle between the vector **i** and the impulse exerted by the bat on the ball,

(2)

(c) the kinetic energy lost by the ball in the impact.

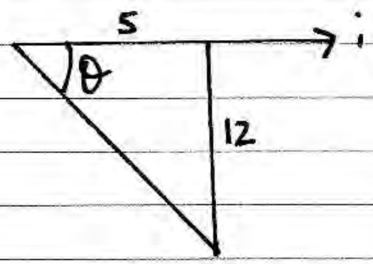
(3)

a) Mom before $= \frac{1}{2}(10i+24j) = 5i+12j$
Mom after $= \frac{1}{2}(20i+0j) = 10i$

Impulse = change in momentum $= 5i-12j$

$|\text{Impulse}| = \sqrt{5^2+12^2} = \underline{13\,Ns}$

b)

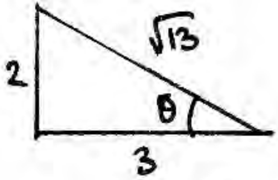


$\theta = \tan^{-1}\left(\frac{12}{5}\right) = 67.4°$ below i

c) Vel before $= \sqrt{10^2+24^2} = 26\ ms^{-1}$
Vel after $= 20\ ms^{-1}$

loss in K.E. $= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}\left(\frac{1}{2}\right)(26^2-20^2) = 69\,J$

6.



$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\theta = \frac{3}{\sqrt{13}}$

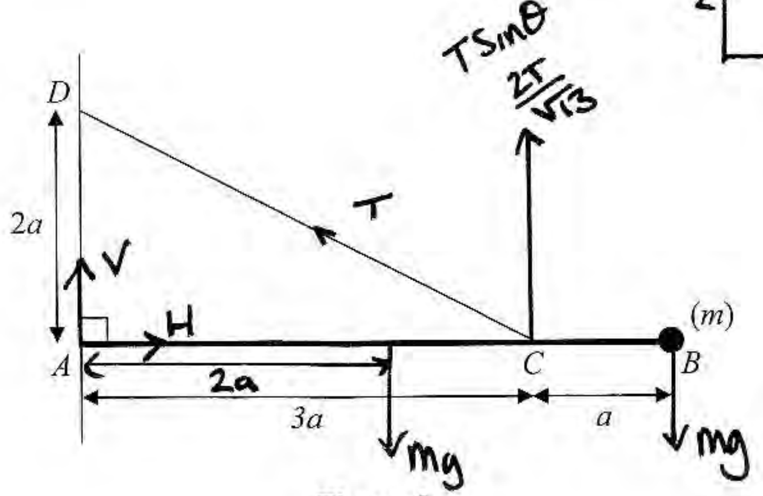


Figure 2

Figure 2 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where $AC = 3a$. The other end of the string is attached to the wall at D, where $AD = 2a$ and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

(a) Show that $T = mg\sqrt{13}$.

(5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leqslant \frac{5}{2}m$.

(3)

a) $A\circlearrowleft \quad mg\times 2\cancel{a} + mg\times 4\cancel{a} = \frac{2T}{\sqrt{13}}\times 3\cancel{a}$

$\cancel{6}mg = \frac{\cancel{6}T}{\sqrt{13}} \Rightarrow T = \sqrt{13}\, mg$

b) $T \leq 2mg\sqrt{13}$

$A\circlearrowleft \quad mg\times 2\cancel{a} + Mg\times 4\cancel{a} \leq 2mg\cancel{\sqrt{13}}\times \frac{2}{\cancel{\sqrt{13}}}\times 3\cancel{a}$

$2mg + 4Mg \leq 12mg$

$\Rightarrow 4M\cancel{g} \leq 10m\cancel{g} \qquad M \leq \frac{5}{2}m$

7.

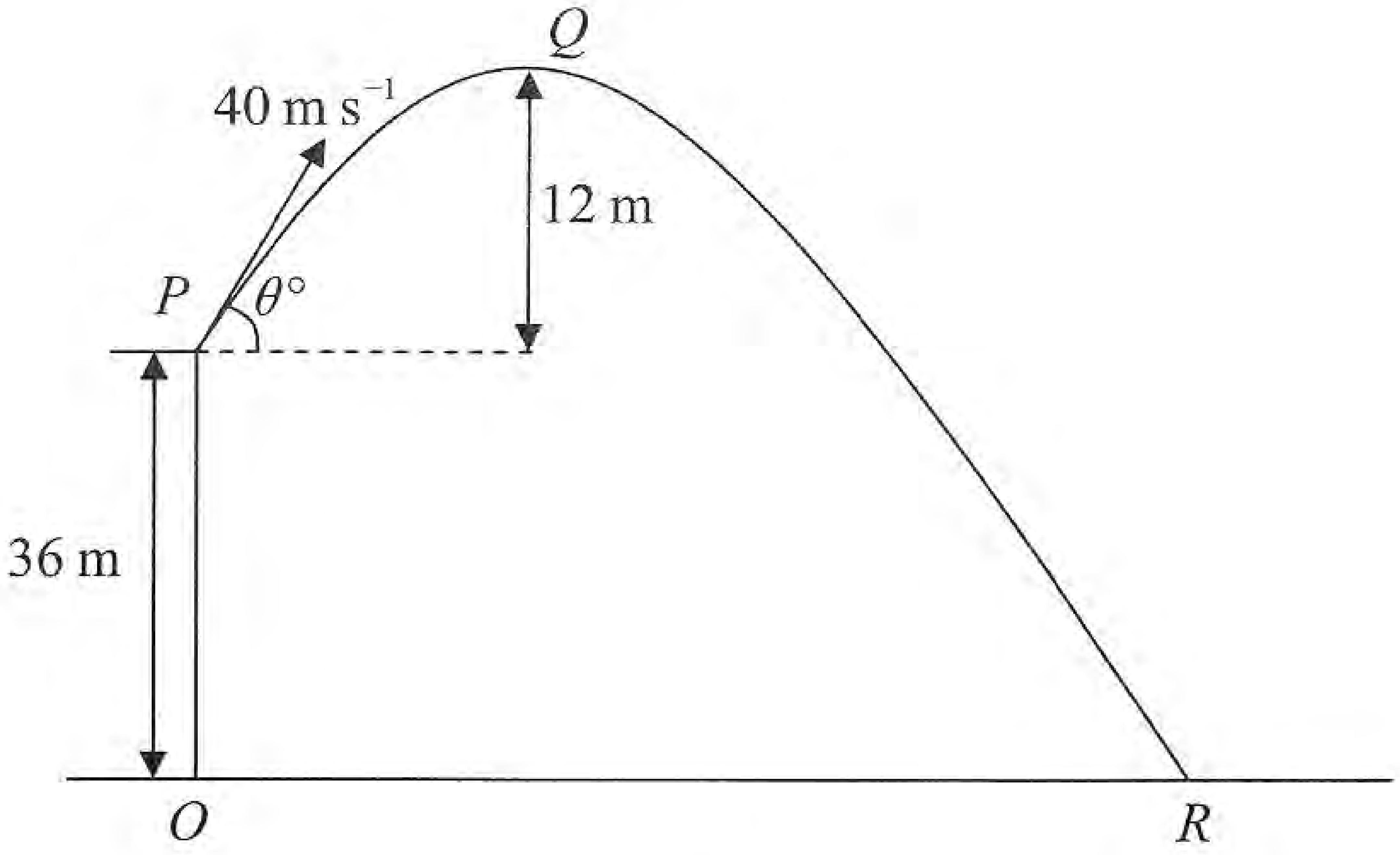


Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in Figure 3. Find

(a) the value of θ,

(3)

(b) the distance OR,

(6)

(c) the speed of the ball as it hits the ground at R.

(3)

a) $u\uparrow = 40\sin\theta$ $\quad v^2 = u^2 + 2as \Rightarrow 0 = (40\sin\theta)^2 - 19.6\times12$

$a = -9.8$

$s = 12$ $\quad 40\sin\theta = \sqrt{19.6\times12}$ $\quad \theta = 22.544805$

$v = 0$ $\quad \theta = 22.5°\ (3sf)$

$u = 15.336231$ $\quad s = ut + \frac{1}{2}at^2 \rightarrow -36 = 15.3..t - 4.9t^2$

$a = -9.8$

$s = -36$ $\quad 4.9t^2 - 15.33..t - 36 = 0$ $\quad t = 4.694...$

$\vec{H}$ Vel $= 40\cos\theta = 36.943...$ dist = Vel × time $OR = 173.4m$

c) $v^2 = u^2 + 2as$ $\quad v^2 = (40\sin 22.54..)^2 - 19.6\times-36 \Rightarrow \downarrow v = 30.672..$

$\vec{v} = 36.943$ $\quad$ Vel $= 36.943i - 30.672j \Rightarrow$ speed $= 48 \text{ ms}^{-1}$

pythag $\Rightarrow$

8. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

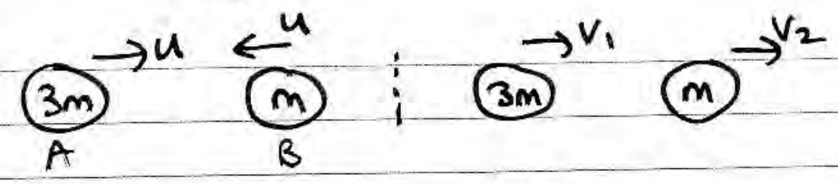
(b) Find the speed of B immediately after hitting the wall.

(2)

The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

(c) Show that $T = \dfrac{112a}{15u}$.

(6)

a) 

$V_2 > V_1$

$e = \dfrac{V_2 - V_1}{2u} = \dfrac{1}{2} \Rightarrow V_2 = u + V_1$

$3mu - mu = 3mV_1 + m(u + V_1) \Rightarrow 2mu = 3mV_1 + mu + mV_1$

$\Rightarrow \cancel{m}u = 4\cancel{m}V_1 \Rightarrow V_1 = \frac{1}{4}u \quad V_2 = u + \frac{1}{4}u = \frac{5}{4}u$

b) $e = \dfrac{\overleftarrow{V}}{\frac{5}{4}\overrightarrow{u}} = \dfrac{2}{5} \Rightarrow \overleftarrow{V} = \frac{2}{5} \times \frac{5}{4}u \quad V = -\frac{1}{2}u$

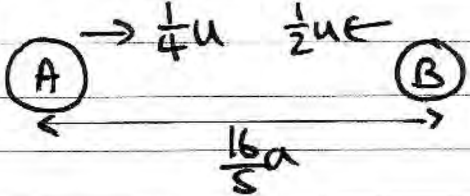
Speed $= \frac{1}{2}u$.

c) (B) Vel $= \frac{5}{4}u$ $S = 4a$ $\quad 4a = \frac{5}{4}u \times t_1 \Rightarrow t_1 = \dfrac{16a}{5u}$

(A) Vel $= \frac{1}{4}u$ $t = \dfrac{16a}{5u}$ $\quad S = \frac{1}{4}u \times \dfrac{16a}{5u} = \dfrac{4a}{5}$

$4a - \frac{4}{5}a = \frac{16}{5}a$

So when B hits the wall A and B are $\frac{16}{5}a$ apart $t = \frac{16a}{5u}$.



speed of approach $= \frac{3}{4}u$

$\frac{16}{5}a = \frac{3}{4}u \times t_2$ $\quad t_2 = \frac{64a}{15u}$

total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{48a + 64a}{15u} = \frac{112a}{15u}$

qed