M2 JAN 13

1. Two uniform rods *AB* and *BC* are rigidly joined at *B* so that $\angle ABC = 90°$. Rod *AB* has length 0.5 m and mass 2 kg. Rod *BC* has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at *G*.

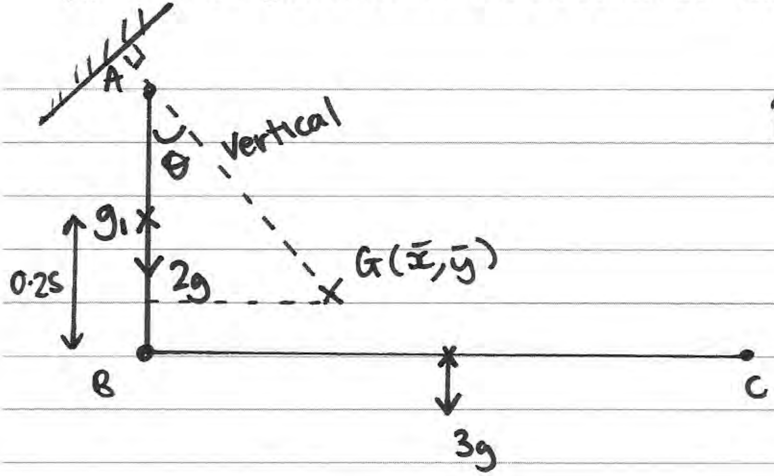
(a) Find the distance of *G* from *BC*.

(2)

The distance of *G* from *AB* is 0.6 m.
The framework is suspended from *A* and hangs freely in equilibrium.

(b) Find the angle between *AB* and the downward vertical at *A*.

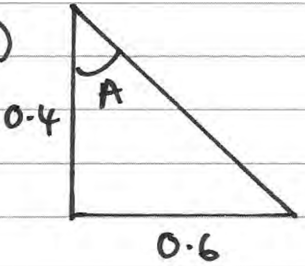
(3)



$$\overset{\frown}{BC} \quad 2\cancel{g} \times 0.25 = 5\cancel{g} \times \bar{y}$$

$$\therefore \ \underline{\bar{y} = 0.1\,\text{m}}$$

b)



$$A = \tan^{-1}\left(\frac{0.6}{0.4}\right) \Rightarrow A = 56.3°$$

2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is 20 m s^{-1}, its acceleration is 0.4 m s^{-2}. The magnitude of the resistance to the motion of the lorry is R newtons.

(a) Find the value of R.

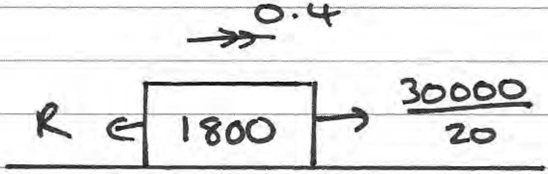
(4)

The lorry now travels up a straight road which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{12}$. The magnitude of the non-gravitational resistance to motion is R newtons. The lorry travels at a constant speed of 20 m s^{-1}.

(b) Find the new rate of working of the lorry's engine.

(5)

a)

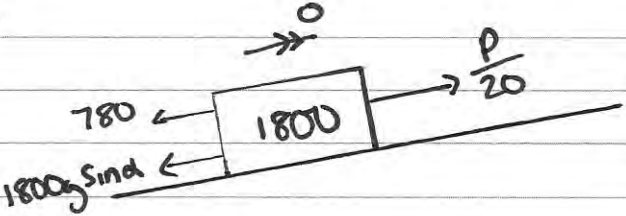


$Rf = ma$

$1500 - R = 1800 \times 04$

$\therefore R = \underline{780N}$

b)



$\frac{P}{20} = 780 + 1800g\left(\frac{1}{12}\right)$

$\frac{P}{20} = 2250$

$\therefore P = \underline{45000W} (45kW)$

3.

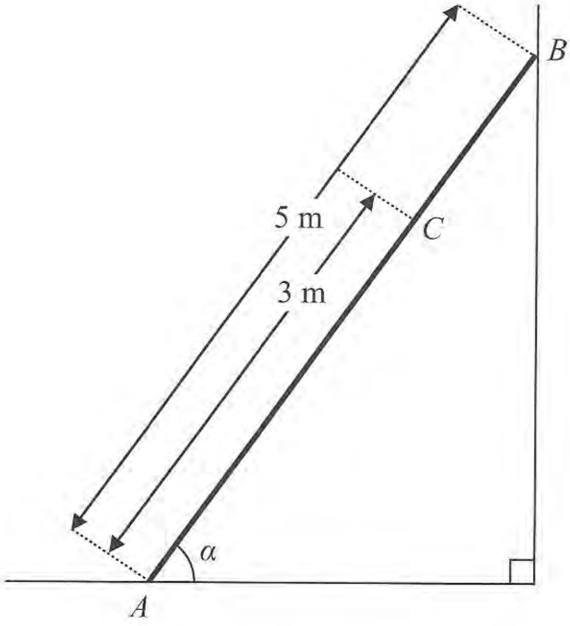
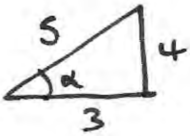


Figure 1

A ladder, of length 5 m and mass 18 kg, has one end *A* resting on rough horizontal ground and its other end *B* resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point *C*, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

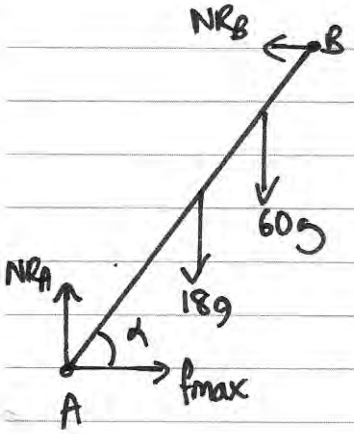
$\tan\alpha = \frac{4}{3}$



$\therefore \quad \sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

(9)



$RF\uparrow = 0 \Rightarrow NRA = 78g$

$f\max = \mu \times NRA = 78g\mu$

$\overrightarrow{RF} = 0 \Rightarrow NRB = 78g\mu.$

$A\circlearrowright \quad 18g \times 2.5\cos\alpha + 60g \times 3\cos\alpha = 78g\mu \times \sin\alpha$

$\Rightarrow 27\cancel{g} + 108\cancel{g} = 312g\mu$

$\therefore \mu = \frac{45}{104}$

4. At time t seconds the velocity of a particle P is $[(4t-5)\mathbf{i}+3\mathbf{j}]\ \text{m s}^{-1}$. When $t=0$, the position vector of P is $(2\mathbf{i}+5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(b) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i}+c\mathbf{j})\ \text{m s}^{-1}$. When $t=0$, the position vector of Q is $(11\mathbf{i}+2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i}+14\mathbf{j})$ m.

(c) Find

(i) the value of c,

(ii) the value of d.

(5)

a) Parallel to $j \Rightarrow i=0 \quad \therefore 4t-5=0 \Rightarrow t=1{\cdot}25$

b) $s=\int v\,dt = \int\begin{pmatrix}4t-5\\3\end{pmatrix}dt = \begin{pmatrix}2t^2-5t+C_1\\3t+C_2\end{pmatrix}$

when $t=0 \quad s=\begin{pmatrix}2\\5\end{pmatrix} \therefore \begin{matrix}C_1=2\\C_2=5\end{matrix} \Rightarrow s=\begin{pmatrix}2t^2-5t+2\\3t+5\end{pmatrix}$

c) $q=\begin{pmatrix}11\\2\end{pmatrix}+t\begin{pmatrix}-2\\c\end{pmatrix}=\begin{pmatrix}11-2t\\2+ct\end{pmatrix}$

$\therefore \begin{pmatrix}2t^2-5t+2\\3t+5\end{pmatrix}=\begin{pmatrix}d\\14\end{pmatrix}=\begin{pmatrix}11-2t\\2+ct\end{pmatrix}$

$\Rightarrow 3t+5=14 \Rightarrow 3t=9 \Rightarrow t=3$

$\Rightarrow \begin{pmatrix}d\\14\end{pmatrix}=\begin{pmatrix}5\\2+3c\end{pmatrix} \quad \therefore d=5$

$14=2+3c \Rightarrow 3c=12 \Rightarrow c=4$

$\underline{c=4}, \underline{d=5}$

5. The point A lies on a rough plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{24}{25}$. A particle P is projected from A, up a line of greatest slope of the plane, with speed U m s^{-1}. The mass of P is 2 kg and the coefficient of friction between P and the plane is $\frac{5}{12}$. The particle comes to instantaneous rest at the point B on the plane, where $AB = 1.5$ m. It then moves back down the plane to A.

(a) Find the work done against friction as P moves from A to B.

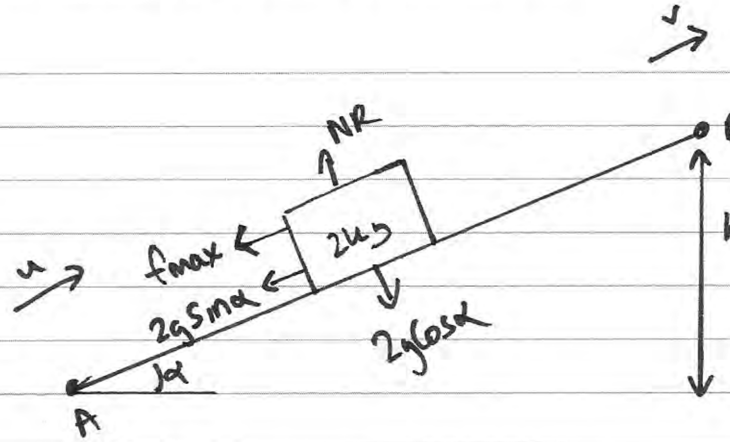
(4)

(b) Use the work-energy principle to find the value of U.

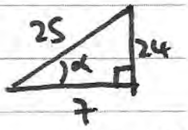
(4)

(c) Find the speed of P when it returns to A.

(3)



$h = 1.5\sin\alpha$
$= 1.44$



$\cos\alpha = \frac{7}{25}$

$\sin\alpha = \frac{24}{25}$

$\uparrow RF = 0 \Rightarrow NR = 2g\cos\alpha = 5.488$

$\mu = \frac{5}{12} \Rightarrow f_{max} = \frac{343}{150}$

$\therefore$ wd against friction $= \frac{343}{150} \times 1.5 = 3.43$ J

b) KE_A − Wd against friction $= PE_B$

$\cancel{\tfrac{1}{2}}(\cancel{2})u^2 - 3.43 = 2g(1.44) \quad \therefore u^2 = 31.654...$

$u = \underline{5.63}$ (3sf)

c) PE_B − Wd against friction $= KE_A$

$2g(1.44) - 3.43 = \cancel{\tfrac{1}{2}}(\cancel{2})v^2 \quad \Rightarrow v^2 = 24.794...$

$v = \underline{4.98 \text{ (3sf)}}$

6.

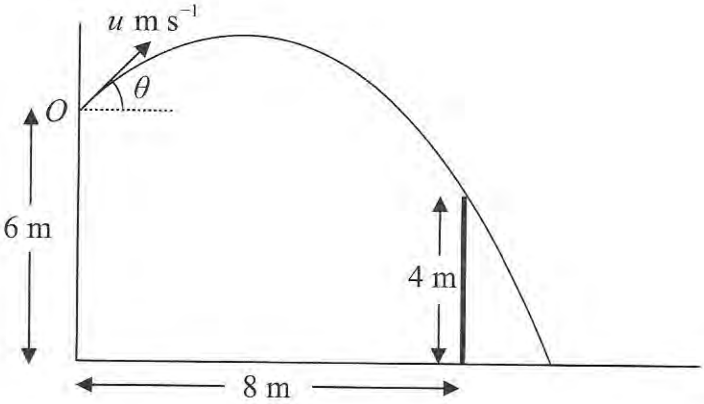


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan\theta = 2.2$

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

$\overrightarrow{H}$ $\quad vel = \frac{dist}{time} \Rightarrow u\cos\theta = \frac{8}{2} \Rightarrow u\cos\theta = 4$

V↑ $\quad S = -2 \qquad S = ut + \frac{1}{2}at^2$

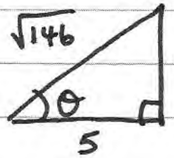
$u = u\sin\theta$

$V =$ $\qquad -2 = 2u\sin\theta - 19.6 \Rightarrow u\sin\theta = 8.8$

$a = -9.8$

$t = 2 \qquad \frac{u\sin\theta}{u\cos\theta} = \frac{8.8}{4} \Rightarrow \tan\theta = 2.2$ #

b) $\tan\theta = 2.2 = \frac{11}{5}$



$\Rightarrow$ $\cos\theta = \frac{5}{\sqrt{146}}$

$\sin\theta = \frac{11}{\sqrt{146}}$

$u\cos\theta = 4 \Rightarrow u \times \frac{5}{\sqrt{146}} = 4 \quad \therefore u = \frac{4\sqrt{146}}{5}$

c) $s = -6$

$u = u\sin\theta = \frac{4\cancel{\sqrt{146}}}{5} \times \frac{11}{\cancel{\sqrt{146}}} = \frac{44}{5} = 8.8$

$v =$

$a = -9.8$

t

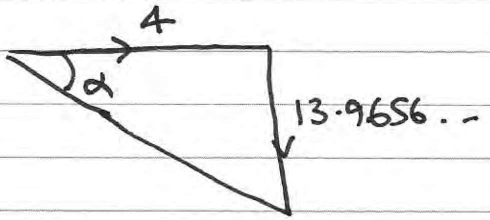
$s = ut + \frac{1}{2}at^2 \Rightarrow -6 = 8.8t - 4.9t^2 \Rightarrow 4.9t^2 - 8.8t - 6 = 0$

$\Rightarrow t = \frac{8.8 \pm \sqrt{8.8^2 - 4(4.9)(-6)}}{9.8}$ $\quad t = 2.323\ldots$

$-0.527\ldots$

$\therefore T = \underline{2.32}\ (3sf)$

d) $v = u + at \Rightarrow V = 8.8 - 9.8 \times 2.32\ldots = -13.9656\ldots$



$\Rightarrow \alpha = \tan^{-1}\left(\frac{13.96\ldots}{4}\right)$

$\therefore \alpha = 74°\ (2sf)$

below horizontal.

7. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(a) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

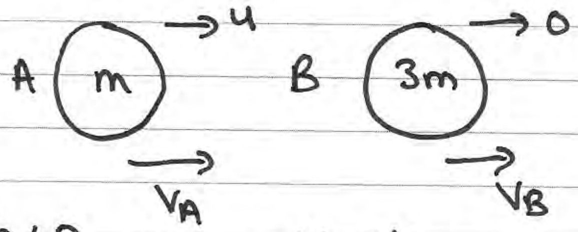
After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(b) find the range of possible values of e,

(6)

(c) determine whether there will be a second collision between A and B.

(3)



$$e = \frac{V_B - V_A}{u}$$

$V_A < 0$ $\quad$ $V_A < V_B$

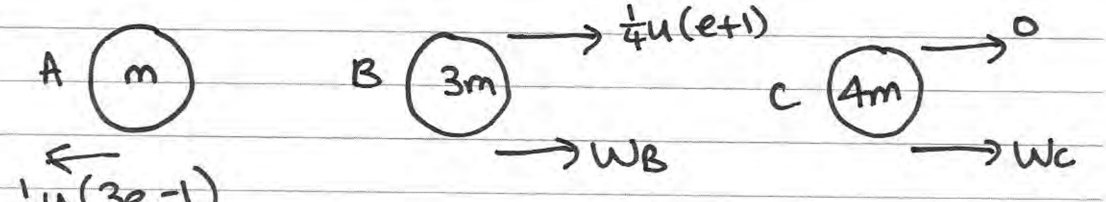
CLM $\Rightarrow$ $\cancel{m}u = \cancel{m}V_A + \cancel{m}3V_B$ $\quad \Rightarrow V_A = u - 3V_B$

$\Rightarrow e = \dfrac{V_B - u + 3V_B}{u}$ $\quad \Rightarrow eu = 4V_B - u$ $\quad \Rightarrow 4V_B = eu + u$

$\therefore$ $V_B = \frac{1}{4}u(e+1)$ $\qquad V_A = u - \frac{3}{4}u(e+1) = \frac{1}{4}u - \frac{3}{4}ue$

$\therefore$ $V_A = \frac{1}{4}u(1-3e)$

b)



$W_B < 0$

$\therefore e > \frac{1}{3}$

$2e = \frac{W_C - W_B}{\frac{1}{4}u(e+1)} \Rightarrow \frac{1}{2}eu(e+1) = W_C - W_B$

CLM $\Rightarrow \frac{3}{4}\cancel{m}u(e+1) = 3\cancel{m}W_B + 4\cancel{m}W_C$

$\therefore \quad 3u(e+1) = 12W_B + 16W_C \quad -$

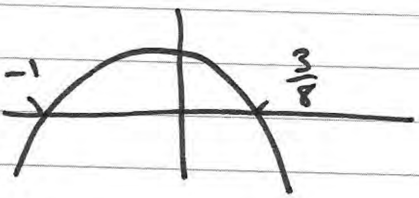
i×16 $\quad 8eu(e+1) = -16W_B + 16W_C$

$u(3-8e)(e+1) = 28W_B$

$\Rightarrow \overrightarrow{W_B} = \frac{1}{28}u(3-8e)(e+1)$

Since $W_B < 0 \Rightarrow (3-8e)(e+1) < 0$

$\frac{3}{8} \quad -1$



$\therefore$ ~~$e < -1$~~ $\quad e > \frac{3}{8}$

$e > \frac{3}{8} \quad e > \frac{1}{3} \quad \therefore e > \frac{3}{8}$

c) speed A $\leftarrow$ $= \frac{1}{4}u(3e-1)$ $\qquad$ Speed B $\leftarrow$ $= \frac{1}{28}u(8e-3)(e+1)$

Collide if $\overleftarrow{\text{Speed B}} > \overleftarrow{\text{Speed A}}$

$\frac{1}{28}\cancel{u}(8e-3)(e+1) > \frac{1}{4}\cancel{u}(3e-1)$

$\Rightarrow \quad (8e-3)(e+1) > 7(3e-1)$

$\Rightarrow 8e^2 + 5e - 3 > 21e - 7$

$\Rightarrow 8e^2 - 16e + 4 > 0$

$$\Rightarrow e^2 - 2e + \tfrac{1}{2} > 0$$

$$\Rightarrow (e-1)^2 - \tfrac{1}{2} > 0 \qquad \Rightarrow (e-1)^2 > \tfrac{1}{2}$$

$$\Rightarrow -\frac{\sqrt{2}}{2} > e - 1 > +\frac{\sqrt{2}}{2}$$

$$\Rightarrow 1 - \frac{\sqrt{2}}{2} > e > 1 + \frac{\sqrt{2}}{2}$$ (not possible!)

$\therefore$ $e < 1 - \frac{\sqrt{2}}{2}$ if they collide

$e < 0.29$ if they collide

but $e > \frac{3}{8}$, $e > 0.375$

$\therefore$ they do <u>not</u> collide.