M2 JAN 06

1a) KE lost $= \frac{1}{2}mu^2 = \frac{1}{2}(3)(8)^2 = \underline{96J}$

b) KE lost = Wd against friction

$\Rightarrow 96 = f_{max} \times 12 \Rightarrow \mu \times NR = 8 \Rightarrow \mu = \frac{8}{3g} = 0.27(2s$

2) $vel = \frac{dr}{dt} = (2t+4)i + (3-3t^2)j$

$t=3,\ v = 10i - 24j \Rightarrow speed = \sqrt{10^2+24^2} = \underline{13\ ms^{-1}}$

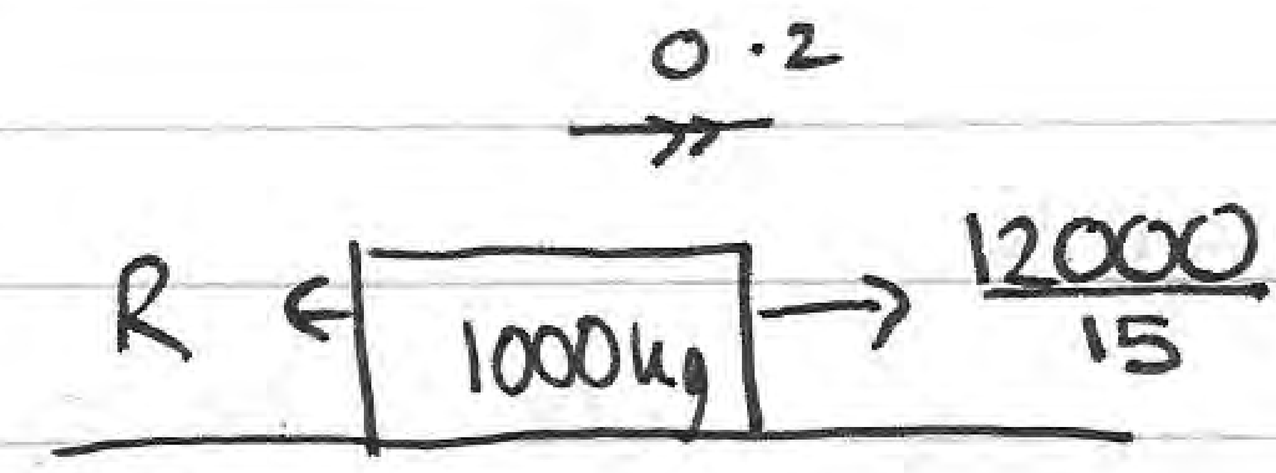
b) Initial momentum $= mu = 0.4(10i-24j) = 4i - 9.6j$

$+$ Impulse $= 8i - 12j$

$=$ final mom $= mv = 12i - 21.6j$

$\div 0.4 \quad v = \underline{30i - 54j}$

3)

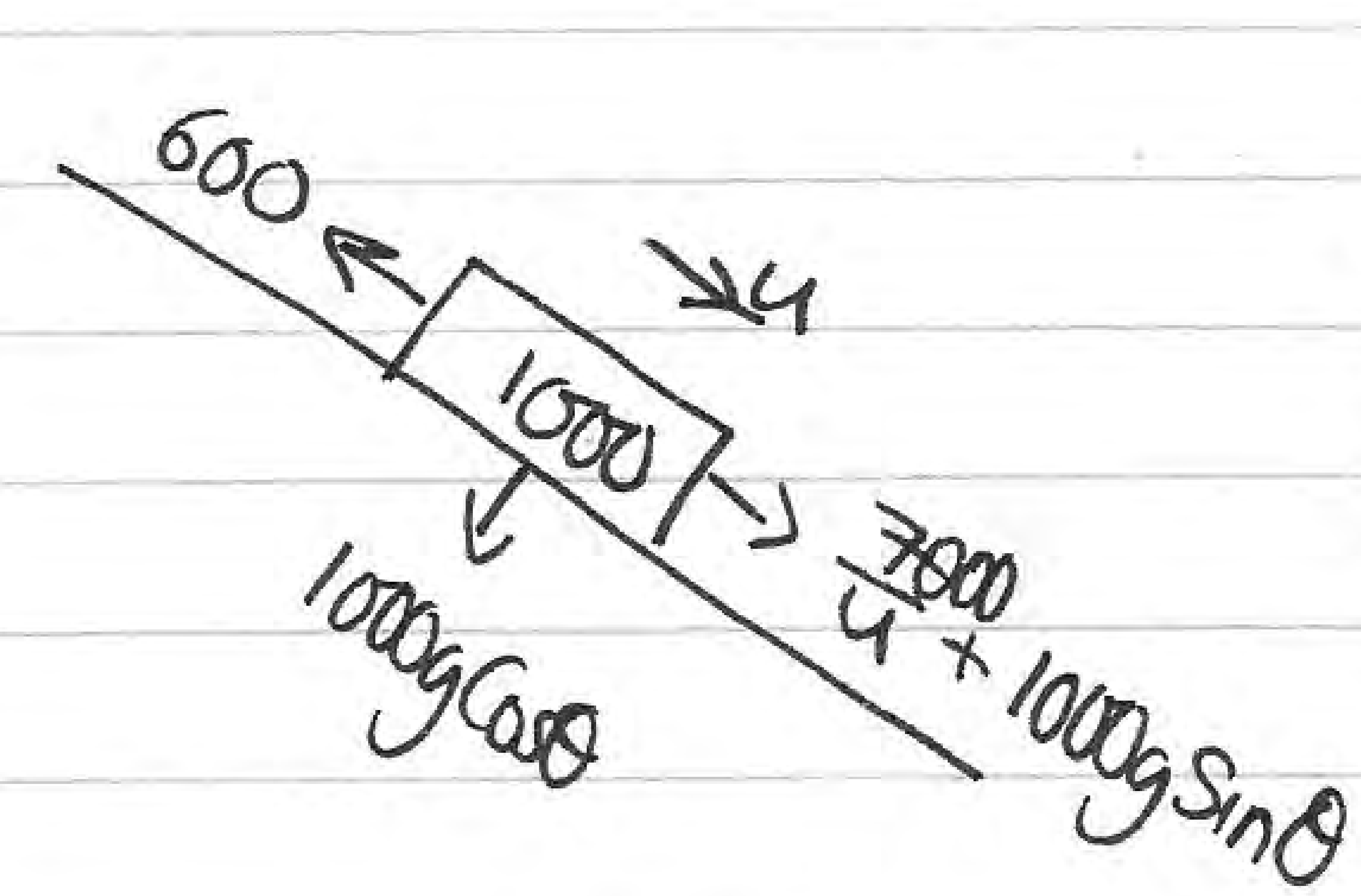


$\overrightarrow{Rf} = ma$

$\Rightarrow \frac{12000}{15} - R = 1000 \times 0.2$

$\Rightarrow 800 - R = 200 \Rightarrow R = 600N$ #

b)



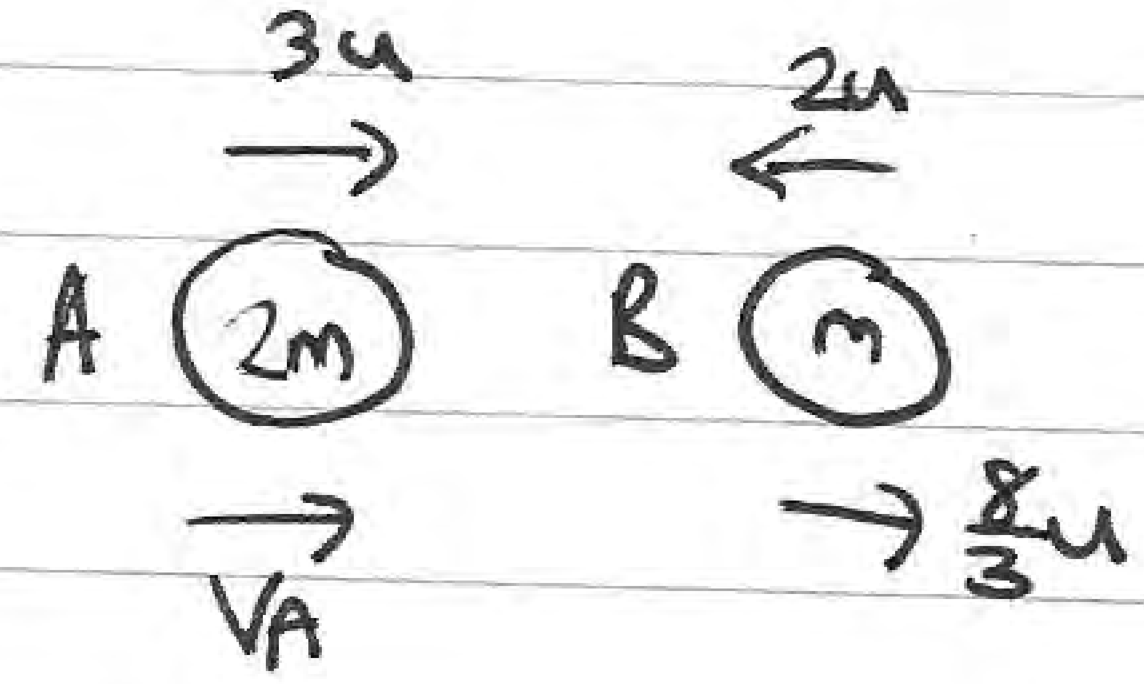
$Rf \downarrow = 0$

$\Rightarrow \frac{7000}{u} + 1000g \times \frac{1}{40} = 600$

$\Rightarrow \frac{7000}{u} = 600 - 25g$

$\Rightarrow u = \frac{7000}{600-25g} \Rightarrow u = \underline{19.7\ ms^{-1}}$ (3sf)

4)



$$\text{CLM} \Rightarrow 2m(3u) + m(-2u) = 2m(V_A) + m\left(\tfrac{8}{3}u\right)$$

$$\Rightarrow 6\cancel{m}u - 2\cancel{m}u = 2\cancel{m}V_A + \tfrac{8}{3}\cancel{m}u$$

$$\Rightarrow \tfrac{4}{3}u = 2V_A \Rightarrow V_A = \tfrac{2}{3}u$$

$$e = \frac{\text{sep}}{\text{app}} = \frac{2\cancel{u}}{5\cancel{u}} \Rightarrow \underline{e = \tfrac{2}{5}}$$

b) total KE before $= \frac{1}{2}(2m)(3u)^2 + \frac{1}{2}m(2u)^2$
$= 11u^2$

total KE after $= \frac{1}{2}(2m)\left(\frac{2}{3}u\right)^2 + \frac{1}{2}(m)\left(\frac{8}{3}u\right)^2$
$= \frac{4}{9}u^2 + \frac{32}{9}u^2 = 4u^2$

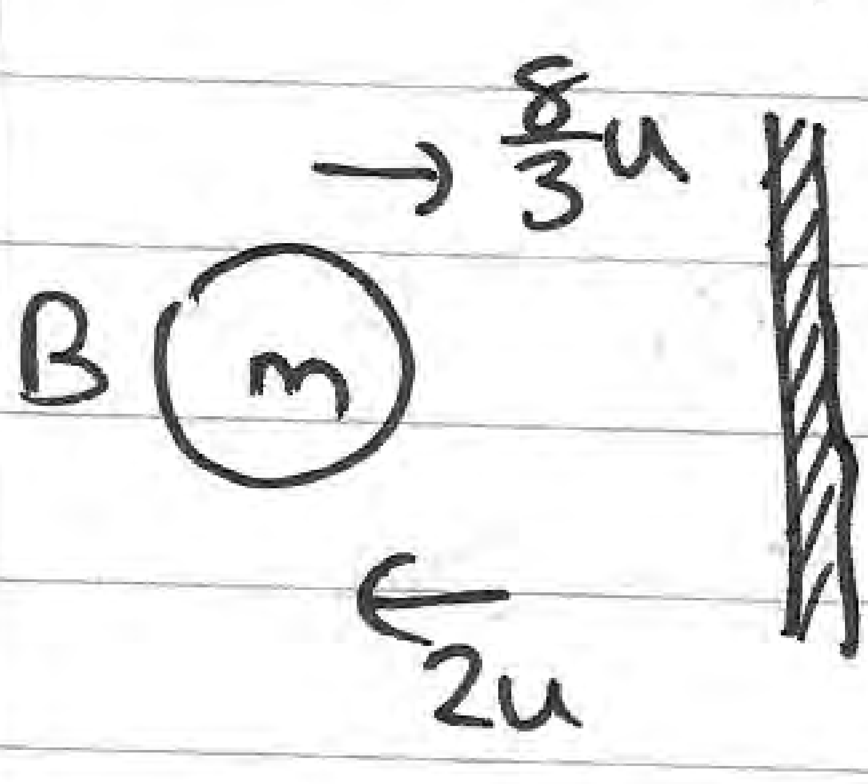
KE lost $= 11u^2 - 4u^2 = \underline{7u^2}$

c) Initial mom $= m\left(\frac{8}{3}u\right) = \frac{8}{3}mu$

$\pm$ Impulse $= \frac{14}{3}mu$ (must be $-$)

final mom $= -\frac{6}{3}mu = -2mu$

$$\Rightarrow \cancel{m}V = -2\cancel{m}u \Rightarrow V = -2u$$



$$e = \frac{\text{sep}}{\text{app}} = \frac{2\cancel{u}}{\frac{8}{3}\cancel{u}} = \frac{6}{8}$$

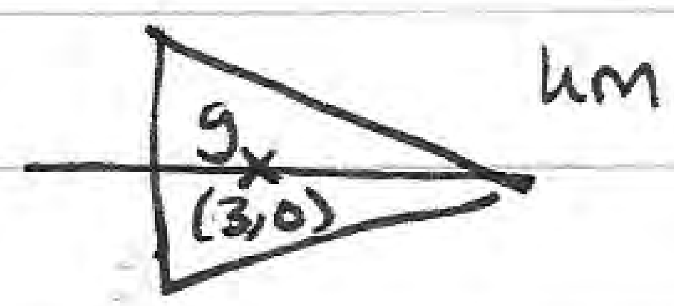
$$\Rightarrow \underline{e = \tfrac{3}{4}}$$

5) $4mg \times 0 + 2mg \times 0 + 6mg \times 9 = 12mg\,\bar{x} \Rightarrow \bar{x} = \frac{54}{12}$

$\bar{x} = 4.5$

x: $2\cancel{mg} \times -4 + 4\cancel{mg} \times 4 + 6\cancel{mg \times 0} = 12\cancel{mg}\,\bar{y}$

$\bar{y} = \frac{8}{12} = \frac{2}{3}$ $\quad (4.5, \frac{2}{3})$



b) $12\cancel{mg} \times 4.5 + k\cancel{mg} \times 3 = (12+k)\cancel{mg} \times 4$

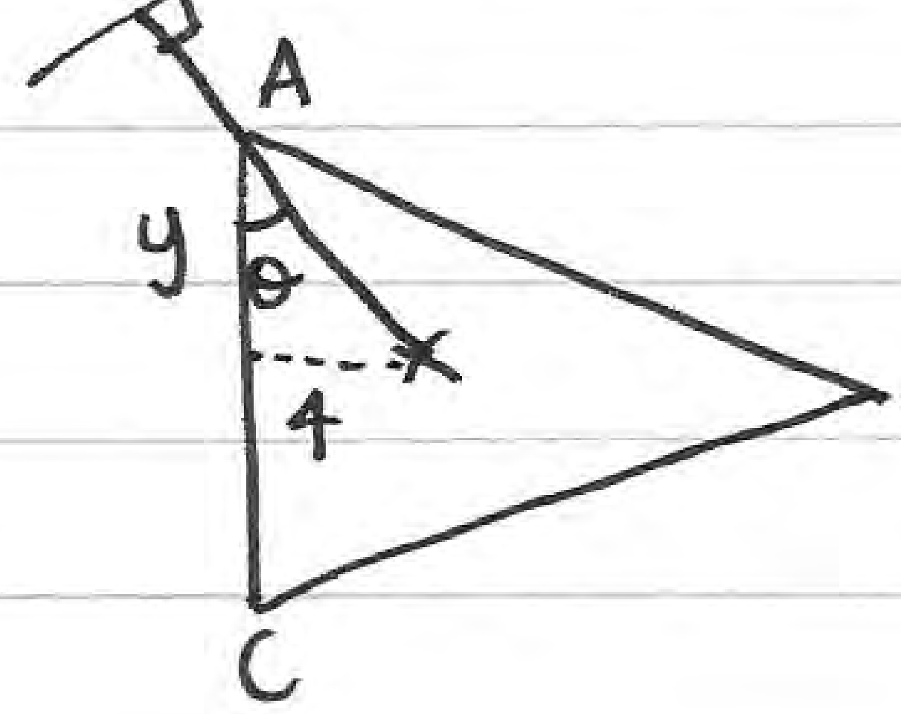
$\Rightarrow 54 + 3k = 4(12+k) \Rightarrow 54 + 3k = 48 + 4k$

$\Rightarrow \underline{k = 6}$ #

c) x: $12\cancel{mg} \times \frac{2}{3} + k\cancel{mg \times 0} = (12+k)\cancel{mg} \times \lambda$

$\Rightarrow 8 = (12+6)\lambda \Rightarrow \lambda = \frac{8}{18} = \frac{4}{9}$

d)

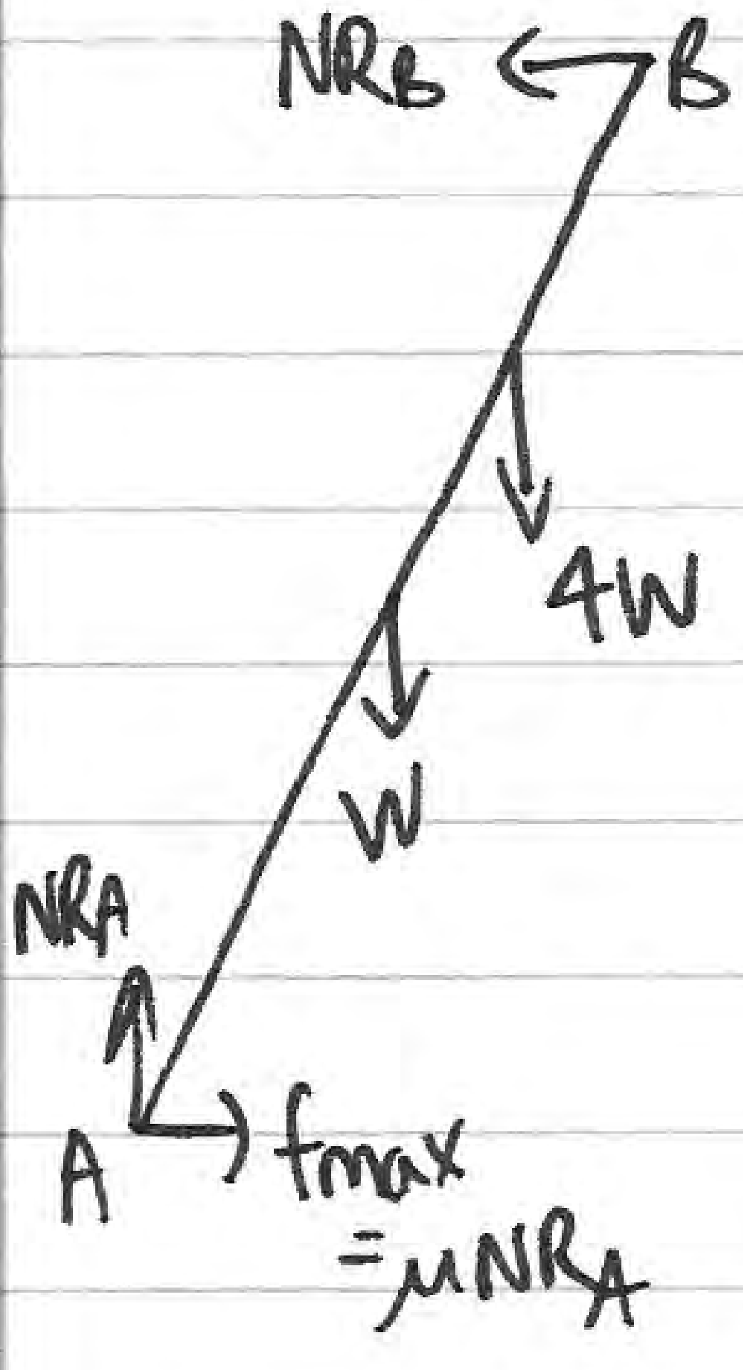


$y = 4 - \frac{4}{9}$

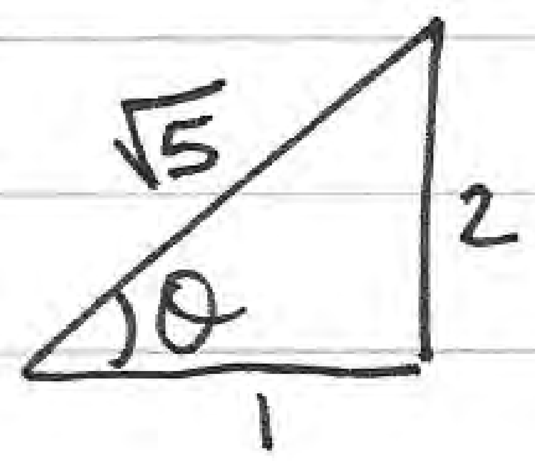
$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{4 - \frac{4}{9}}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{9}{8}\right) = \cancel{48.4}°$

83.7°

6)



$\tan\theta = \frac{2}{1}$



$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$R_f\uparrow = 0 \Rightarrow N_{RA} = 5W$

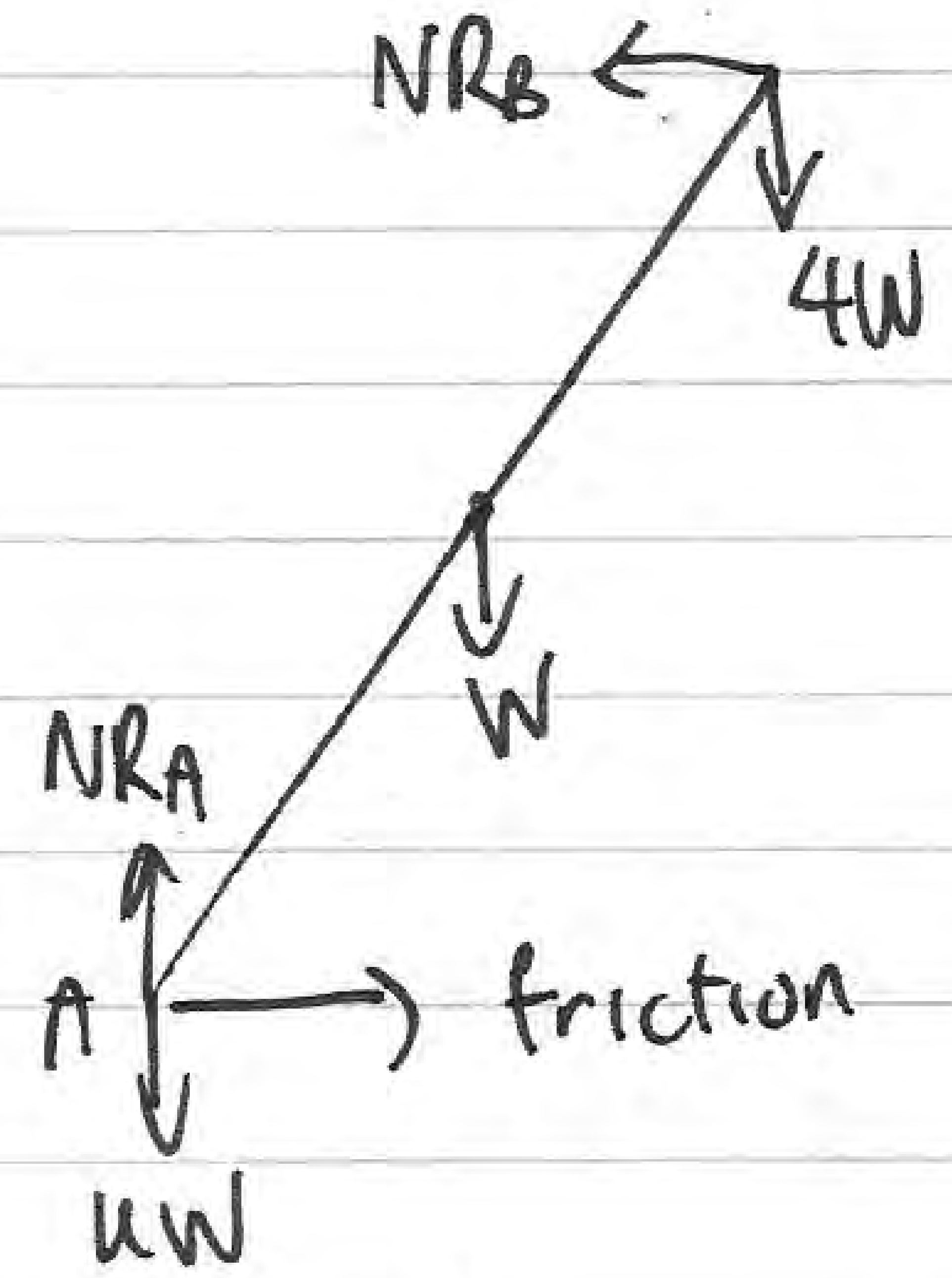
$\vec{R_f} = 0 \quad f_{max} = N_{RB} \Rightarrow \mu N_{RA} = N_{RB} \Rightarrow \mu = \frac{N_{RB}}{5W}$

A↺ $W \times 2\cancel{a} \times \left(\frac{1}{\cancel{\sqrt{5}}}\right) + 4W \times 3\cancel{a} \times \left(\frac{1}{\cancel{\sqrt{5}}}\right) = N_{RB} \times 4\cancel{a} \times \left(\frac{2}{\cancel{\sqrt{5}}}\right)$

$2W + 12W = 8N_{RB} \qquad N_{RB} = \frac{14}{8}W$

$\therefore \quad \mu = \dfrac{\frac{14}{8}W}{5W} = \dfrac{14}{40} = \dfrac{7}{20}$ #

b)



$R_f \uparrow = 0 \Rightarrow NR_A = (5+k)W$

$\overrightarrow{R_f} = 0 \Rightarrow$ friction $= NR_B$

$A \circlearrowright \; W \times 2a\left(\frac{1}{\sqrt{5}}\right) + 4W \times 4a\left(\frac{1}{\sqrt{5}}\right) = NR_B \times 4a\left(\frac{2}{\sqrt{5}}\right)$

$\Rightarrow 2W + 16W = 8NR_B \quad \Rightarrow NR_B = \frac{9}{4}W$

$f_{max} = \mu NR_A = \frac{7}{20}(5+k)W \qquad \text{friction} = NR_B = \frac{9}{4}W$

friction $\leq f_{max} \Rightarrow \frac{9}{4}\cancel{W} \leq \frac{7}{20}(5+k)\cancel{W}$

$\Rightarrow \frac{45}{7} \leq 5+k \quad \Rightarrow k \geq \frac{10}{7}$

7) $\overrightarrow{H}$ Vel $= 11\cos 30 \quad$ dist $= 10 \quad \Rightarrow t = \dfrac{10}{11\frac{\sqrt{3}}{2}} = 1.05$

b) $V\uparrow$ $u\uparrow = 11\sin 30 = 5.5$

$a\uparrow = -9.8$

$t = \frac{20\sqrt{3}}{33}$

$S = ut + \frac{1}{2}at^2$

$S = 5.5\left(\frac{20\sqrt{3}}{33}\right) - 4.9\left(\frac{20\sqrt{3}}{33}\right)^2$

$S = 0.374\ldots$ above A

$\Rightarrow C = 1 - 0.374\ldots$ below T

$\therefore C$ is 0.63m below T

c) $u\uparrow = V\sin 30 = \frac{1}{2}V$

$a = -9.8$

$S = 1$ (above A)

$\overrightarrow{H}$ Vel $= V\cos 30 \quad x = 10$

$\Rightarrow t = \dfrac{10}{V\frac{\sqrt{3}}{2}} = \dfrac{20\sqrt{3}}{3V}$

$\Rightarrow 1 = \left(\frac{1}{2}\cancel{V}\right)\left(\frac{20\sqrt{3}}{3\cancel{V}}\right) - 4.9\left(\frac{20\sqrt{3}}{3V}\right)^2$

$\Rightarrow 1 - \frac{10\sqrt{3}}{3} = \frac{1960}{3v^2}$

$\Rightarrow (3 - 10\sqrt{3})v^2 = 1960 \quad \Rightarrow V = \sqrt{\frac{1960}{3-10\sqrt{3}}}$

$V = 11.7\,ms^{-1}$ (3sf)

d) We have ignored wind, air resistance, spin. We have also considered the ball to be a particle, so v could be increased and decreased but still hit the target as the ball might hit the target at the top or bottom of the ball.