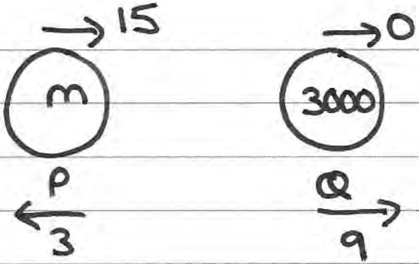
M1 JAN 12

1. A railway truck P, of mass m kg, is moving along a straight horizontal track with speed $15\,\text{m s}^{-1}$. Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is $3\,\text{m s}^{-1}$ and the speed of Q is $9\,\text{m s}^{-1}$. The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

(a) the magnitude of the impulse exerted by P on Q,
(2)

(b) the value of m.
(3)



Mom Q before = 0
Mom Q after = 27000
$\Rightarrow$ Impulse = 27000 Ns

b) Conservation of momentum $\Rightarrow$

$15m = -3m + 27000 \Rightarrow 18m = 27000$

$\Rightarrow m = 1500$ kg

2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

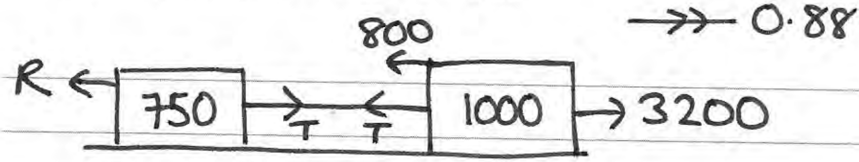
Given that the acceleration of the car and the caravan is $0.88\,\mathrm{m\,s^{-2}}$,

(a) show that $R = 860$,

(3)

(b) find the tension in the tow-bar.

(3)



a) whole system $\overrightarrow{RF} = ma$

$3200 + T - T - 800 - R = (750 + 1000) \times 0.88$

$\Rightarrow 2400 - R = 1540 \Rightarrow R = 2400 - 1540 = \underline{860\,\text{N}}$ #

b) Caravan $\overrightarrow{RF} = ma$

$T - 860 = 750 \times 0.88 \Rightarrow T = \underline{1520\,\text{N}}$

3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\ \text{N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\ \text{N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\ \text{N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

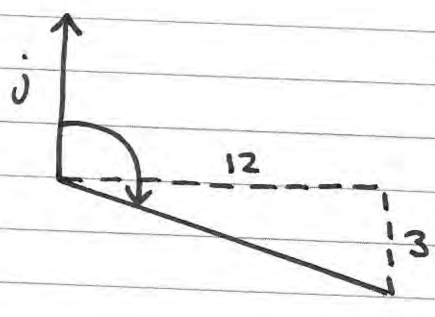
a) $F_1 + F_2 + F_3 = 0 \Rightarrow \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow p = -12 , q = 3$

b) $R_f = \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ $|R| = \sqrt{12^2 + 3^2}$

$|R| = \underline{12.4\ \text{N}}$ (3sf)

c)



angle $= 90 + \tan^{-1}\left(\frac{3}{12}\right)$

$= \underline{104°}$ (n.d.)

4.

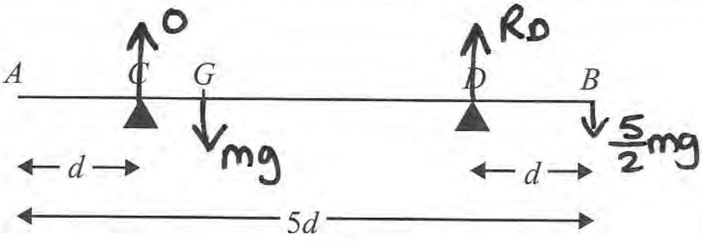


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D. $\Rightarrow R_C = 0$

(a) Show that $GD = \frac{5}{2}d$. ?

(4)

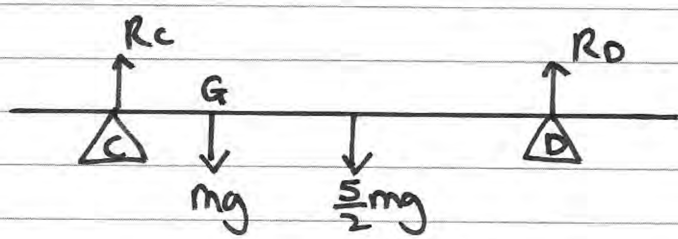
The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5) ?

a) $D \curvearrowright \quad \frac{5}{2}\cancel{mg} \times d = \cancel{mg} \times GD \quad \Rightarrow GD = \frac{5}{2}d$

b) $\cancel{R(\uparrow) = 0 \Rightarrow R_D = mg + \frac{5}{2}mg = \frac{7}{2}mg}$ read the question ☺



$CG = 3d - \frac{5}{2}d$
$= \frac{1}{2}d$

$C \curvearrowright \quad mg \times \frac{1}{2}d + \frac{5}{2}mg \times \frac{3}{2}d = R_D \times 3d$

$\frac{1}{2}mgd + \frac{15}{4}mgd = R_D \times 3d$

$\frac{17}{4}mg\cancel{d} = R_D \times 3\cancel{d} \quad \Rightarrow R_D = \underline{\frac{17}{12}mg}$

5. A stone is projected vertically upwards from a point A with speed $u\,\mathrm{m\,s^{-1}}$. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

a) $s = 0$ $\quad s = ut + \frac{1}{2}at^2$

u

v $\quad 0 = 3\frac{4}{7}u - 4.9(3\frac{4}{7})^2$

$a = -9.8$

$t = 3\frac{4}{7}$ $\quad \cancel{3\frac{4}{7}}u = 4.9(3\frac{4}{7})^{\cancel{2}}$

$u = 4.9 \times 3\frac{4}{7} = 17.5$ #

b) s

$u = 17.5$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = 17.5^2 - 19.6s$

$a = -9.8$

t $\quad \Rightarrow s = 15.625 \simeq 15.6\text{m (3sf)}$

c) $s = 6.6$ $\quad s = ut + \frac{1}{2}at^2$

$u = 17.5$

v $\quad 6.6 = 17.5t - 4.9t^2$

$a = -9.8$

t $\quad 4.9t^2 - 17.5t + 6.6 = 0$

$49t^2 - 175t + 66 = 0$

$(7t - 3)(7t - 22) = 0$

$t_1 = \frac{3}{7} \quad t_2 = \frac{22}{7}$

time above $= \frac{19}{7}$ sec

6. A car moves along a straight horizontal road from a point A to a point B, where $AB = 885$ m. The car accelerates from rest at A to a speed of $15\,\mathrm{m\,s^{-1}}$ at a constant rate $a\,\mathrm{m\,s^{-2}}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of $15\,\mathrm{m\,s^{-1}}$ for T seconds. The car then decelerates at a constant rate of $2.5\,\mathrm{m\,s^{-2}}$ stopping at B.

(a) Find the time for which the car decelerates. **(2)**

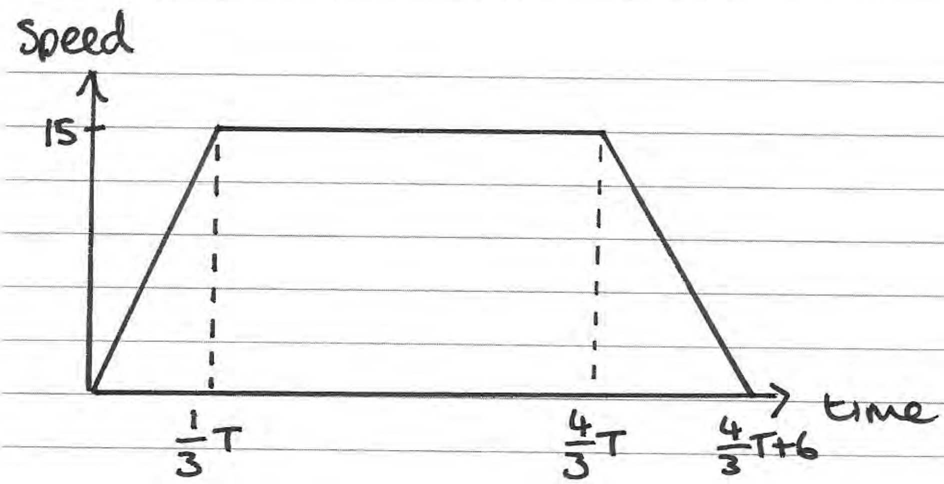
$\frac{15}{2.5} = 6$ sec

(b) Sketch a speed-time graph for the motion of the car. **(2)**

(c) Find the value of T. **(4)**

(d) Find the value of a. **(2)**

(e) Sketch an acceleration-time graph for the motion of the car. **(3)**

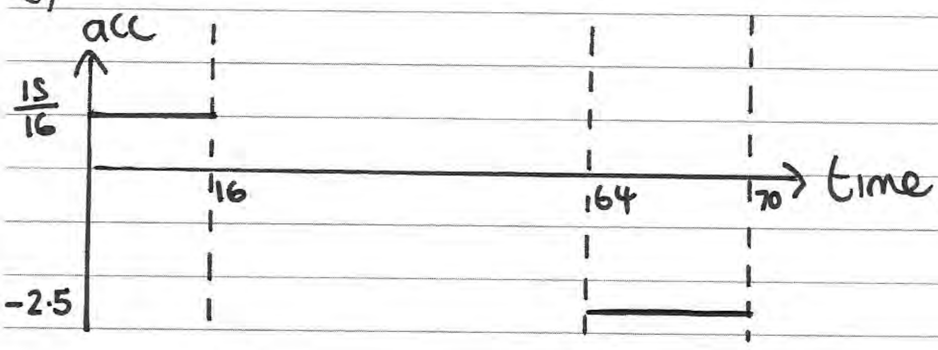


c) $\left(\frac{1}{6}T \times 15\right) + (T \times 15) + (3 \times 15) = 885$

$17\frac{1}{2}T = 840 \Rightarrow T = \underline{48 \text{ sec}}$

d) $a = \frac{15}{16}\ \mathrm{ms^{-2}}$

e)



7. *[In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.]*

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P. $\sqrt{4^2+8^2} = 8.94$ km/h (3sf) **(2)**

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t. $p = \begin{pmatrix} 2 \\ -8 \end{pmatrix} + t\begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2-4t \\ 8t-8 \end{pmatrix}$ **(1)**

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j}) \quad = \begin{pmatrix} 18-6t \\ 12-8t \end{pmatrix}$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

c) due west $\Rightarrow$ j component equal

$8t-8 = 12-8t \Rightarrow 16t = 20 \quad t = \frac{5}{4}$

d) when $t = \frac{5}{4}$ i component of $P = 2-4(\frac{5}{4}) = -3$

j component of $q = 18-6(\frac{5}{4}) = 10.5$

$\therefore$ distance between P and Q $= 13.5$ km

8.

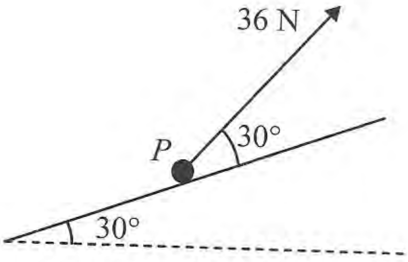


Figure 2

A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of $16\,\mathrm{m\,s^{-1}}$ under the action of a force of magnitude 36 N. The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P, and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ. Find

(a) the magnitude of the normal reaction between P and the plane,

(4)

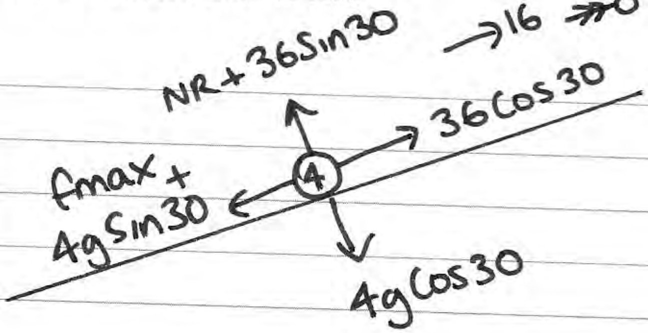
(b) the value of μ.

(5)

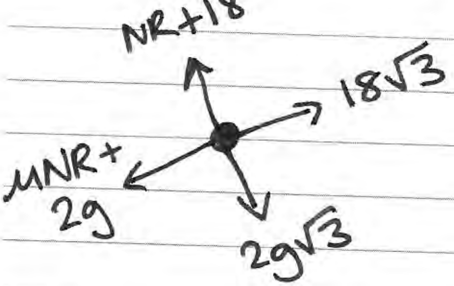
The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest.

(5)



Constant speed
$\Rightarrow$ acc = 0
$\Rightarrow$ equilibrium

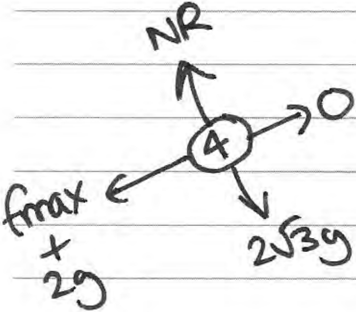


$R_f \uparrow = 0 \Rightarrow NR = -18 + 2\sqrt{3}g$

$\simeq 15.9\,\mathrm{N}$ (3sf)

$R_f \nearrow = 0 \Rightarrow \mu NR = 18\sqrt{3} - 2g$

$\therefore \mu = \dfrac{18\sqrt{3} - 2g}{-18 + 2\sqrt{3}g} \simeq 0.726$ (3sf)



$$NR = 2\sqrt{3}g$$

$$f_{max} = \mu \times 2\sqrt{3}g = 24.6432\ldots$$

$$\overrightarrow{RF} = ma$$

$$-24.6432\ldots - 2g = 4a$$

$$\Rightarrow a = -11.0608\ldots$$

s
$u = 16$
$v = 0$
$a = -11.0608\ldots$
t

$$v^2 = u^2 + 2as$$

$$0 = 256 - 22.1216\ldots s$$

$$\therefore s \simeq \underline{11.6\text{m}} \ (3\text{sf})$$