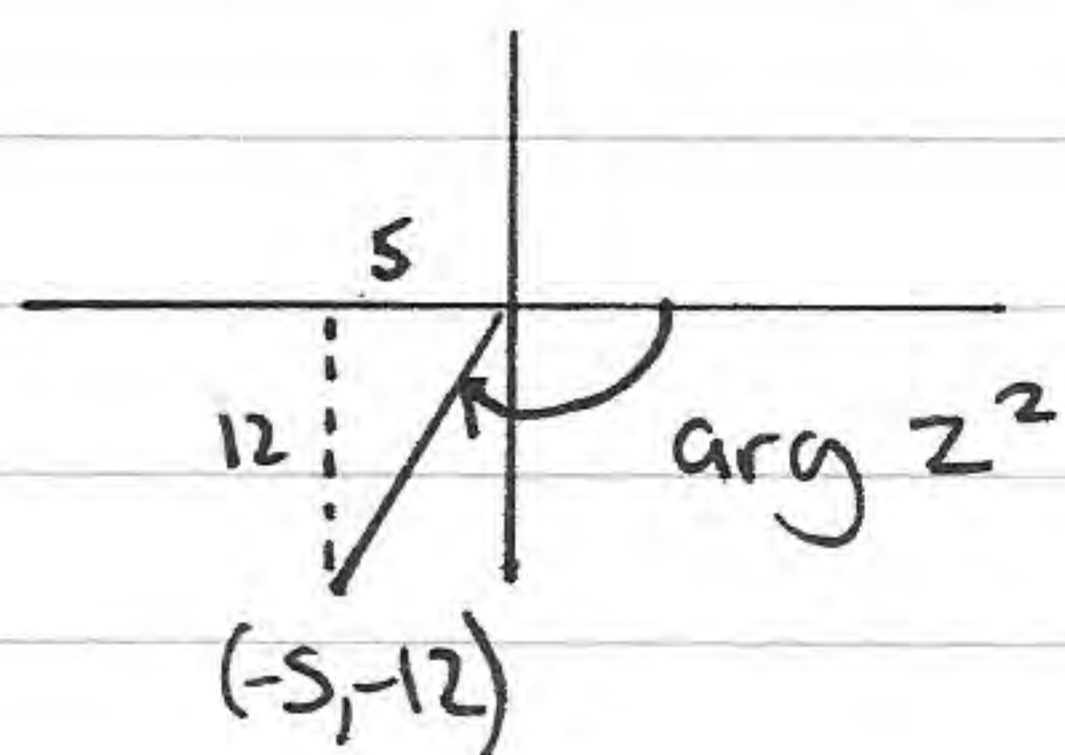


FPI June 2010

$$1) z^2 = (2-3i)(2-3i) = 4 - 6i - 6i + 9i^2 = (4-9) - 12i = -5 - 12i \neq$$

$$b) |z^2| = \sqrt{5^2 + 12^2} = \underline{13}$$

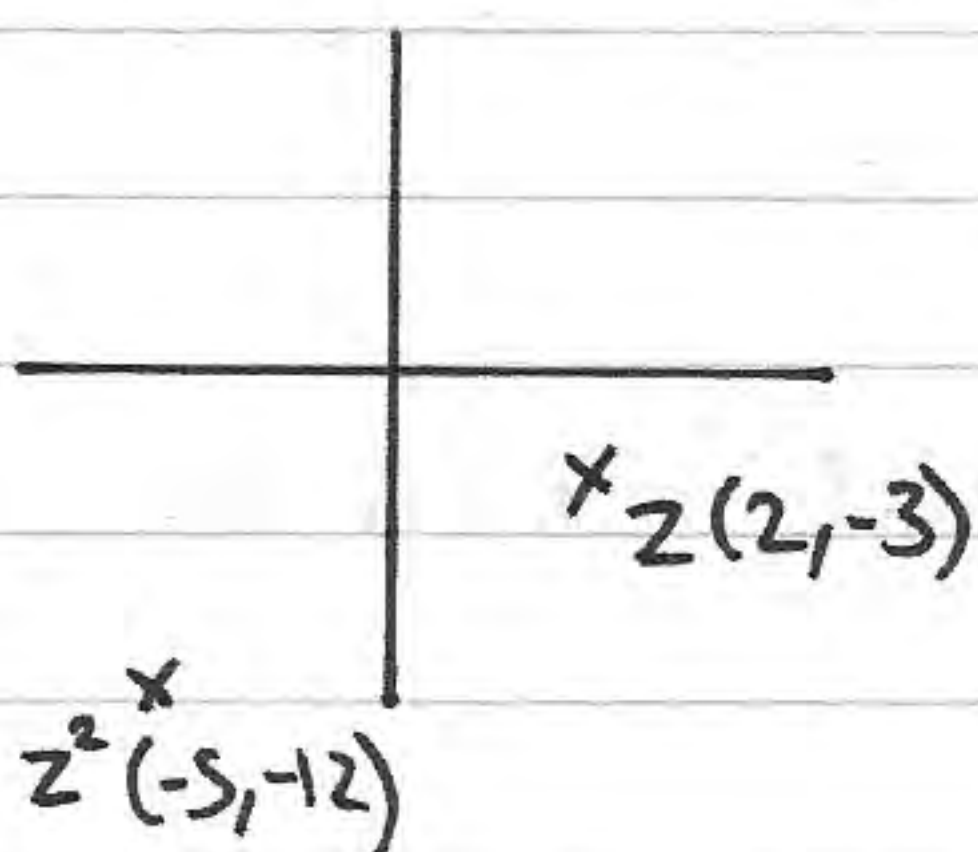
c)



$$\arg(z^2) = -\left(\pi - \tan^{-1}\left(\frac{12}{5}\right)\right)$$

$$\arg(z^2) = \underline{-1.97^\circ}$$

d)



$$2) M = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix} \quad a=2 \Rightarrow M = \begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$$

$$\det M = 4 \times 2 - 3 \times 6 = -10 \quad \Rightarrow M^{-1} = \frac{-1}{10} \begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix}$$

$$b) \text{ Singular if } \det M = 0 \Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9 \Rightarrow \underline{a = \pm 3}$$

$$3) f(x) = x^3 - \frac{7}{x} + 2 \quad f(1.4) = -0.256 < 0$$
$$f(1.5) = +0.708 > 0$$

change of sign $\Rightarrow x \in [1.4, 1.5]$

b)	a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
	1.4	-0.26	1.5	0.71	1.45	0.22
	1.4	-0.26	1.45	0.22	1.425	-0.02
	1.425	-0.02	1.45	0.22		

$$\Rightarrow \alpha \in [1.425, 1.45]$$

$$c) f'(x) = 3x^2 + 7x^{-2}$$

$$x_0 = 1.45 \quad x_1 = 1.45 - \frac{f(1.45)}{f'(1.45)} = \underline{1.427}$$

$$4) f(x) = (x+3)(x^2+ax+b) = x^3+x^2+44x+150$$

$$3b = 150 \Rightarrow b = 50$$

$$3x^2 + ax^2 = 1x^2 \Rightarrow 3+a = 1 \Rightarrow a = -2$$

$$b) f(x) = (x+3)(x^2-2x+50)$$

$$x^2-2x+50=0 \Rightarrow (x-1)^2-1+50=0$$

$$\Rightarrow (x-1)^2 = -49$$

$$\Rightarrow x-1 = \pm 7i$$

$$\Rightarrow x = 1 \pm 7i$$

$$\alpha = -3, 1+7i, 1-7i$$

$$c) -3 + 1+7i + 1-7i = \underline{-1}$$

5) $y^2 = 20x = 4ax \Rightarrow a = 5$ focus $(5, 0)$ directrix $x + 5 = 0$

a) $P(5t^2, 10t)$ $y^2 = 20(5t^2) = 100t^2 \Rightarrow y = \sqrt{100t^2} = 10t$ #

b) $A(80, 40)$ $S(5, 0)$ $M_{SA} = \frac{40}{75} = \frac{8}{15}$

6a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$

d) $AB = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} u & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6u + c & 0 \\ 4u + 2c & -8 \end{pmatrix}$

e) $6u + c = 8 \Rightarrow 6u + c = 8$
 $4u + 2c = 0 \Rightarrow 2u + c = 0$ -
 $\underline{4u = 8} \Rightarrow \underline{u = 2} \Rightarrow \underline{c = -4}$

6) $f(u) = 2^u + 6^u$
 $\Rightarrow f(u+1) = 2^{u+1} + 6^{u+1} = 2(2^u) + 6(6^u)$
 $= 6(2^u) - 4(2^u) + 6(6^u)$
 $= 6(2^u + 6^u) - 4(2^u)$
 $= 6f(u) - 4(2^u)$ #

b) $n=1$ $f(1) = 2^1 + 6^1 = 8 = 8 \times 1$ hence \div by 8
 $n=2$ $f(2) = 2^2 + 6^2 = 40 = 8 \times 5$ hence \div by 8

$n=u$ $f(u) = 2^u + 6^u$ assume is \div by 8

$n=u+1$ $f(u+1) = 6f(u) - 4(2^u) = 6(8\lambda) - 4 \times 2(2^{u-1})$
 $= 8(6\lambda - (2^{u-1}))$
hence \div by 8

true for $n=1$, true for $n=u+1$ if true for $n=u$
 \therefore by induction true for all $n \in \mathbb{Z}^+$ #

$$7) \quad xy = c^2 \quad (ct, \frac{c}{t}) \quad ct = 3c \Rightarrow t = 3$$

$$y = \frac{c}{t} = \frac{c}{3}$$

$$b) \quad y = c^2 x^{-1} \Rightarrow \frac{dy}{dx} = -c^2 x^{-2} = -\frac{c^2}{x^2} \Rightarrow M_t = \frac{-c^2}{x^2}$$

$$M_t = -\frac{1}{t^2} \Rightarrow M_n = t^2$$

$$y - \frac{c}{3} = t^2(x - ct) \quad t = 3 \Rightarrow y - \frac{c}{3} = 9(x - 3c)$$

$$\textcircled{\times 3} \Rightarrow 3y - c = 27(x - 3c) \Rightarrow 3y - c = 27x - 81c$$

$$\Rightarrow 3y = 27x - 80c \neq$$

$$c) \quad xy = c^2 \Rightarrow x \left(\frac{27x - 80c}{3} \right) = c^2$$

$$\Rightarrow 27x^2 - 80cx - 3c^2 = 0 \quad xc = 3c \text{ is a root}$$

$$\Rightarrow (27x + c)(x - 3c) \Rightarrow x = -\frac{c}{27} \Rightarrow t = -\frac{1}{27}$$

$$B\left(-\frac{c}{27}, -27c\right)$$

$$y = -27c$$

$$8) \quad n=1 \quad \sum_1^1 r^2 = 1^2 = 1 \quad \frac{1}{6}(1)(2)(3) = 1 \therefore \text{true for } n=1$$

$$n=2 \quad \sum_1^2 r^2 = 1^2 + 2^2 = 5 \quad \frac{1}{6}(2)(3)(5) = 5 \therefore \text{true for } n=2$$

$$n=k \quad \sum_1^k r^2 = \frac{1}{6}k(k+1)(2k+1) \quad \text{assume to be true}$$

$$n=k+1 \quad \sum_1^{k+1} r^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$$

$$\begin{aligned}
\sum_1^{u+1} r^2 &= \sum_1^u r^2 + (u+1)^2 = \frac{1}{6}u(u+1)(2u+1) + (u+1)^2 \\
&= \frac{1}{6}(u+1)[u(2u+1) + 6(u+1)] \\
&= \frac{1}{6}(u+1)[2u^2 + 7u + 6] \\
&= \frac{1}{6}(u+1)(u+2)(2u+3) \text{ as required.}
\end{aligned}$$

\therefore true for $n=1$, true for $n=u+1$ if true for $n=u$
so by induction true for all $n \in \mathbb{Z}$.

b)

$$\begin{aligned}
\sum (r+2)(r+3) &= \sum r^2 + 5\sum r + \sum 6 \\
&= \frac{1}{6}n(n+1)(2n+1) + 5\left(\frac{1}{2}n(n+1)\right) + 6n \\
&= \frac{1}{6}n(n+1)(2n+1) + \frac{15}{6}n(n+1) + \frac{36}{6}n \\
&= \frac{1}{6}n[(n+1)(2n+1) + 15(n+1) + 36] \\
&= \frac{1}{6}n[2n^2 + 3n + 1 + 15n + 15 + 36] \\
&= \frac{1}{6}n[2n^2 + 18n + 52] \\
&= \frac{1}{3}n[n^2 + 9n + 26] \quad a=9 \quad b=26.
\end{aligned}$$

c)

$$\begin{aligned}
\sum_{n+1}^{2n} (r+2)(r+3) &= \frac{1}{3}(2n)[(2n)^2 + 9(2n) + 26] - \frac{1}{3}n(n^2 + 9n + 26) \\
&= \frac{1}{3}n[2(4n^2 + 18n + 26) - (n^2 + 9n + 26)] \\
&= \frac{1}{3}n[7n^2 + 27n + 26] \quad \#
\end{aligned}$$