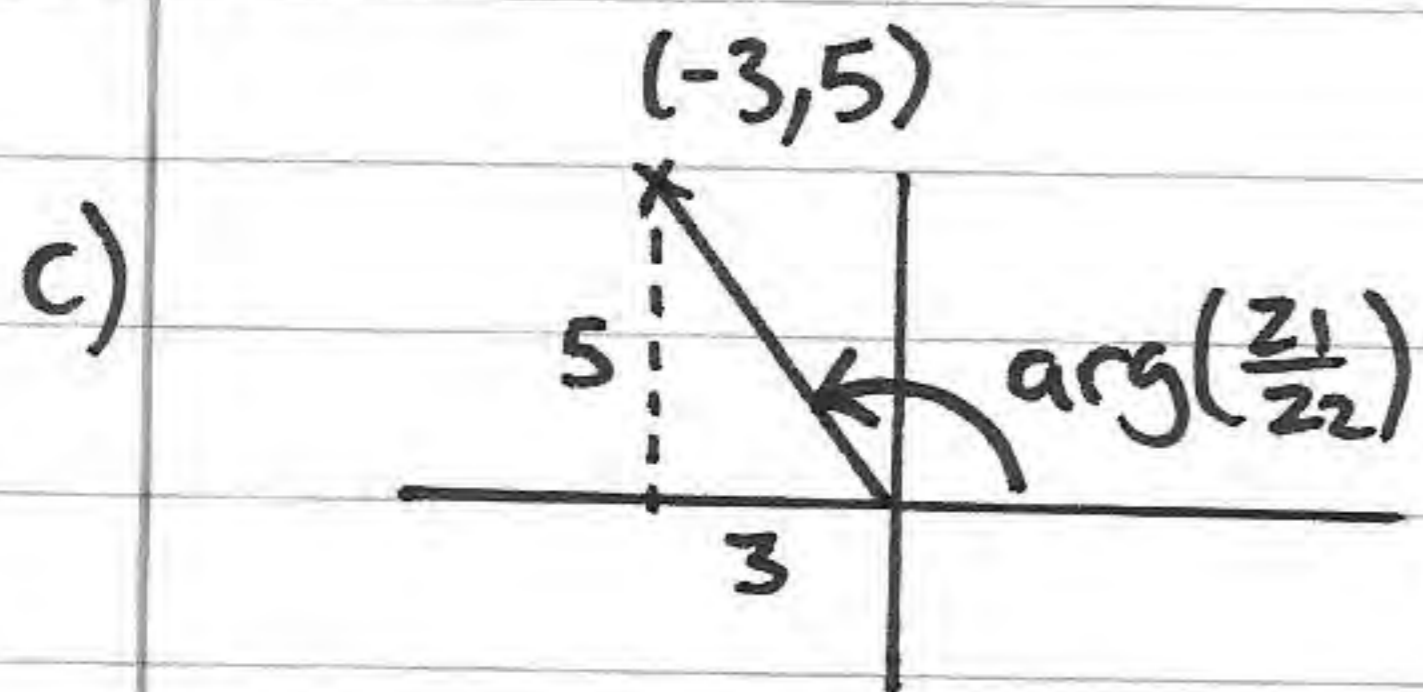


FP1 JAN 2010

$$1) \frac{z_1}{z_2} = \frac{2+8i}{1-i} \frac{(1+i)}{(1+i)} = \frac{2+2i+8i+8i^2}{1-i^2} = \frac{-6+10i}{2}$$

$$\frac{z_1}{z_2} = \underline{-3+5i}$$

$$b) \left| \frac{z_1}{z_2} \right| = \sqrt{3^2+5^2} = \sqrt{34}$$



$$c) \arg\left(\frac{z_1}{z_2}\right) = \pi - \tan^{-1}\left(\frac{5}{3}\right) = \underline{2.11^c}$$

$$2) f(x) = 3x^2 - \frac{11}{x^2} \quad \begin{array}{l} f(1.3) = -1.439 \\ f(1.4) = 0.268 \end{array}$$

b)

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1.3	-1.4	1.4	0.3	1.35	-0.57
1.35	-0.6	1.4	0.3	1.375	-0.15
1.375	-0.15	1.4	0.3		

$$\Rightarrow \alpha \in [1.375, 1.4]$$

$$c) f'(x) = 6x + 22x^{-3}$$

$$x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} = 1.384$$

$$3) \quad n=1 \quad u_1 = 2 \quad u_1 = 5^{1-1} + 1 = 1 + 1 = 2 \quad \therefore \text{true for } n=1$$

$$n=2 \quad u_2 = 5(2) - 4 = 6 \quad u_2 = 5^{2-1} + 1 = 5 + 1 = 6 \quad \therefore \text{true for } n=2$$

$$n=k \quad u_k = 5(u_{k-1}) - 4 \quad u_k = 5^{k-1} + 1 \quad \text{assume true}$$

$$n=k+1 \quad u_{k+1} = 5(u_k) - 4 = 5(5^{k-1} + 1) - 4$$

$$= 5^k + 5 - 4$$

$$= 5^k + 1$$

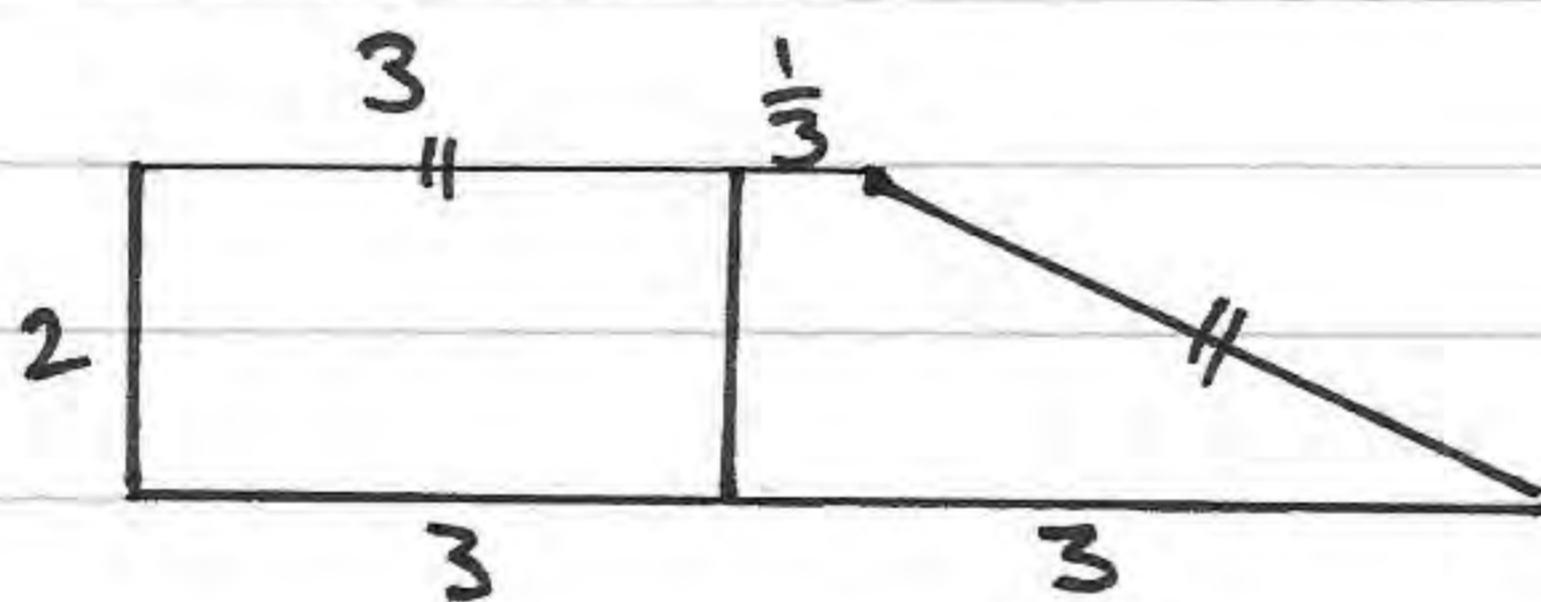
$$u_{k+1} = 5^{(k+1)-1} + 1 = 5^k + 1 \quad \text{as required.}$$

true for $n=1$, true for $n=k+1$ if true for $n=k$
 \therefore by induction true for all $n \in \mathbb{Z}^+$

$$4) \quad y^2 = 12x = 4ax \Rightarrow a=3 \quad \text{focus } (3,0) \quad \text{directrix } x+3=0$$

$$a) \quad S(3,0)$$

$$b) \quad A(-3,0)$$



$$x = \frac{1}{3} \Rightarrow y^2 = 4$$

$$\Rightarrow P\left(\frac{1}{3}, 2\right) \Rightarrow B(-3, 2)$$

$$\text{Perimeter } r = 2 \times \left(3 \frac{1}{3}\right) + 2 + 6 = \frac{44}{3}$$

$$5) \quad A = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix} \quad \det A = a(a+4) + 10 = a^2 + 4a + 10$$

$$b) \quad \det A = (a+2)^2 - 4 + 10 = (a+2)^2 + 6 \Rightarrow \det A \geq 6$$

hence non-singular

$$c) \quad a=0 \quad A = \begin{pmatrix} 0 & -5 \\ 2 & 4 \end{pmatrix} \quad \det A = 10 \quad A^{-1} = \frac{1}{10} \begin{pmatrix} 4 & 5 \\ -2 & 0 \end{pmatrix}$$

6) $5-2i$

b) $(x-2)(x-(5+2i))(x-(5-2i))$

Sum = 10

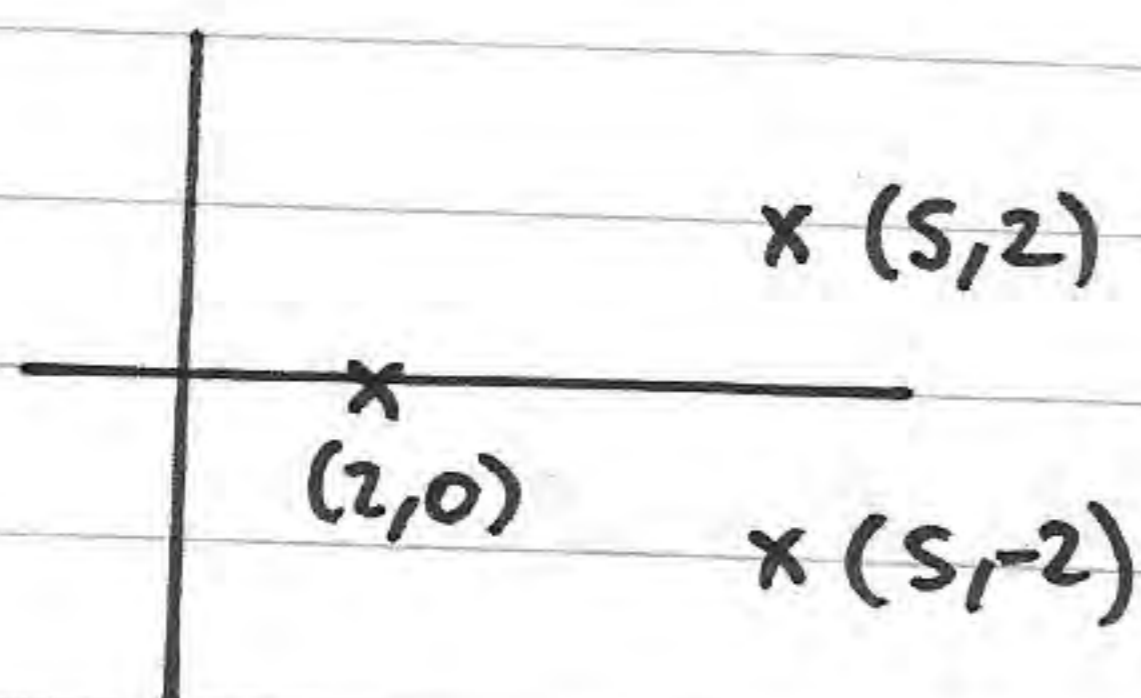
product = $25-4i^2$
= 29

$\Rightarrow (x-2)(x^2-10x+29)$

$d = -2 \times 29 = \underline{-58}$

$c = -2 \times -10 + 29 = \underline{49}$

c)



7) $y = c^2 x^{-1} \Rightarrow \frac{dy}{dx} = -c^2 x^{-2} = \frac{-c^2}{x^2} = \frac{-c^2}{c^2 t^2} = -\frac{1}{t^2}$

$\Rightarrow M_t = -\frac{1}{t^2} \Rightarrow y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$

$(x t^2) \quad y t^2 - ct = -x + ct$

$\Rightarrow t^2 y + x = 2ct \quad \#$

b) $(15c, -c) \Rightarrow t^2(-c) + (15c) = 2ct$

$\Rightarrow ct^2 + 2ct - 15c = 0$

$\Rightarrow t^2 + 2t - 15 = 0$

$\Rightarrow (t+5)(t-3) = 0 \quad t = -5, t = 3$

$(ct, \frac{c}{t}) \Rightarrow \underline{A(-5c, -\frac{c}{5})} \quad \underline{B(3c, \frac{c}{3})}$

$$8) \quad n=1 \quad \sum_1^1 r^3 = 1^3 = 1 \quad \frac{1}{4}(1)^2(1+1)^2 = \frac{1}{4} \times 1 \times 2^2 = 1 \therefore \text{true}$$

$$n=2 \quad \sum_1^2 r^3 = 1^3 + 2^3 = 9 \quad \frac{1}{4}(2)^2(2+1)^2 = \frac{1}{4} \times 4 \times 9 = 9 \therefore \text{true}$$

$$n=k \quad \sum_1^k r^3 = \frac{1}{4}k^2(k+1)^2 \quad \text{assume true}$$

$$n=k+1 \quad \sum_1^{k+1} r^3 = \frac{1}{4}(k+1)^2(k+2)^2$$

$$\sum_1^{k+1} r^3 = \sum_1^k r^3 + (k+1)^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

$$= \frac{1}{4}(k+1)^2(k^2 + 4(k+1))$$

$$= \frac{1}{4}(k+1)^2(k^2 + 4k + 4)$$

$$= \frac{1}{4}(k+1)^2(k+2)^2 \quad \text{as required.}$$

true for $n=1$, true for $n=k+1$ if true for $n=k$
 \therefore true for all $n \in \mathbb{Z}^+$ by induction.

$$b) \sum_{r=1}^n r^3 + 3r + 2 = \sum_{r=1}^n r^3 + 3 \sum_{r=1}^n r + \sum_{r=1}^n 2$$

$$= \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right) + 2n$$

$$= \frac{1}{4}n \left[n(n+1)^2 + 6n(n+1) + 8 \right]$$

$$= \frac{1}{4}n \left(n^3 + 2n^2 + n + 6n + 6 + 8 \right)$$

$$= \frac{1}{4}n \left(n^3 + 2n^2 + 7n + 14 \right)$$

$$= \frac{1}{4}n \left(n^2(n+2) + 7(n+2) \right)$$

$$= \frac{1}{4}n(n+2)(n^2+7) \quad \#$$

$$c) \sum_{15}^{25} r^3 + 3r + 2 = \frac{1}{4}(25)(27)(25^2+7) - \frac{1}{4}(14)(16)(14^2+7)$$

$$= 95282$$

$$9) M = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{rotation } 45^\circ \text{ anticlockwise about the origin}$$

$$b) \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$$

$$\Rightarrow \frac{1}{\sqrt{2}}p - \frac{1}{\sqrt{2}}q = 3\sqrt{2} \quad (\times\sqrt{2}) \quad p - q = 6$$

$$\Rightarrow \frac{1}{\sqrt{2}}p + \frac{1}{\sqrt{2}}q = 4\sqrt{2} \quad (\times\sqrt{2}) \quad p + q = 8$$

$$2p = 14$$

$$p = 7 \quad q = 1$$

$$c) \quad OA = \sqrt{7^2 + 1^2} = \sqrt{50} = \underline{5\sqrt{2}}$$

$$d) \quad M^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$e) \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} -4\sqrt{2} \\ 3\sqrt{2} \end{pmatrix} \quad C(\underline{-4\sqrt{2}, 3\sqrt{2}})$$