

FPI Jan 09

$$1) \quad 2x^3 - 8x^2 + 7x - 3 = (x-3)(2x^2 + Ax + 1)$$

$$Ax^2 - 6x^2 = -8x^2 \quad \Rightarrow \quad A = -2$$

$$\Rightarrow (x-3)(2x^2 - 2x + 1)$$

$$2) \quad \sum 6r^2 + 4r - 1 = 6 \sum r^2 + 4 \sum r - \sum 1$$

$$= 6 \left(\frac{1}{6} n(n+1)(2n+1) \right) + 4 \left(\frac{1}{2} n(n+1) \right) - n$$

$$= n \left[(n+1)(2n+1) + 2n + 2 - 1 \right]$$

$$= n \left[2n^2 + 5n + 2 \right] = n(2n+1)(n+2) \quad \#$$

$$b) \quad \sum_{r=1}^{20} 6r^2 + 4r - 1 = 20(41)(22) - 10(21)(12)$$

$$= \underline{15520}$$

$$3) y = \frac{5}{t} = \frac{25}{5t} \Rightarrow y = \frac{25}{x} \Rightarrow xy = 25$$

$$b) A(5,5) \quad B(25,1) \quad \text{Midpoint } AB = (15,3)$$

$$4) n=1 \quad \sum_1^1 \frac{1}{r(r+1)} = \frac{1}{1(2)} = \frac{1}{2} \quad \frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2} \quad \text{tr}$$

$$n=k \quad \sum_1^k \frac{1}{r(r+1)} = \frac{k}{k+1}$$

$$n=k+1 \quad \sum_1^{k+1} \frac{1}{r(r+1)} = \frac{k+1}{k+2}$$

$$\sum_1^{k+1} \frac{1}{r(r+1)} = \sum_1^k \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2} \quad \text{as required.}$$

true for $n=1$, true for $n=k$ if true for $n=k+1$
 \therefore by induction true for all $n \in \mathbb{Z}^+$.

$$5) \quad f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20 \quad f(1.1) = 0.31 > 0$$

$$f(1.2) = -0.28 < 0$$

Sign change $\Rightarrow \alpha \in [1.1, 1.2]$

$$b) \quad f(x) = 3x^{\frac{1}{2}} + 18x^{-\frac{1}{2}} - 20 \Rightarrow f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$$

$$c) \quad x_1 = 1.1 - \frac{f(1.1)}{f'(1.1)} = \underline{1.15}$$

$$6) \quad u_1 = 6 \quad u_{n+1} = 6u_n - 5 \quad u_n = 5(6^{n-1}) + 1$$

$$u_1 = 6$$

$$u_1 = 5(6^{1-1}) + 1 = 5 + 1 = 6$$

$$u_2 = 6(6) - 5 = 31$$

$$u_2 = 5(6^{2-1}) + 1 = 30 + 1 = 31$$

$$n = k \quad u_k = 5(6^{k-1}) + 1$$

$$n = k+1 \quad u_{k+1} = 6u_k - 5 = 6(5(6^{k-1}) + 1) - 5$$

$$= 5(6 \times 6^{k-1}) + 6 - 5$$

$$= 5(6^k) + 1$$

$$u_{k+1} = 5(6^{k+1-1}) + 1 = 5(6^k) + 1 \text{ as required.}$$

true for $n=1$, true for $n=2$, true for $n=k+1$ if true for $n=k$
 \therefore by induction true for all $n \in \mathbb{Z}^+$

$$7) \quad X = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix} \quad \det X = -2 + a \Rightarrow X^{-1} = \frac{1}{a-2} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$$

$$b) \quad \frac{1}{a-2} \begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \frac{-1}{a-2} + 2 = 1$$

$$\frac{-1}{a-2} = -1 \Rightarrow a-2 = 1$$

$$\underline{a=3}$$

$$8) \quad y^2 = 4ax \quad \frac{d}{dx} y^2 = \frac{d}{dx} (4ax) \Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow m_t = \frac{2a}{2aq} = \frac{1}{q}$$

$$y - 2aq = \frac{1}{q}(x - aq^2)$$

$$\textcircled{xq} \quad qy - 2aq^2 = x - aq^2 \Rightarrow yq = x + aq^2 \neq$$

$$b) \quad \text{meets } y \text{ when } x=0 \Rightarrow yq = aq^2 \Rightarrow y = aq$$

$$R(0, aq) \text{ perp} \Rightarrow m = -q \quad y - aq = -q(x - 0)$$

$$\Rightarrow y = -qx + aq$$

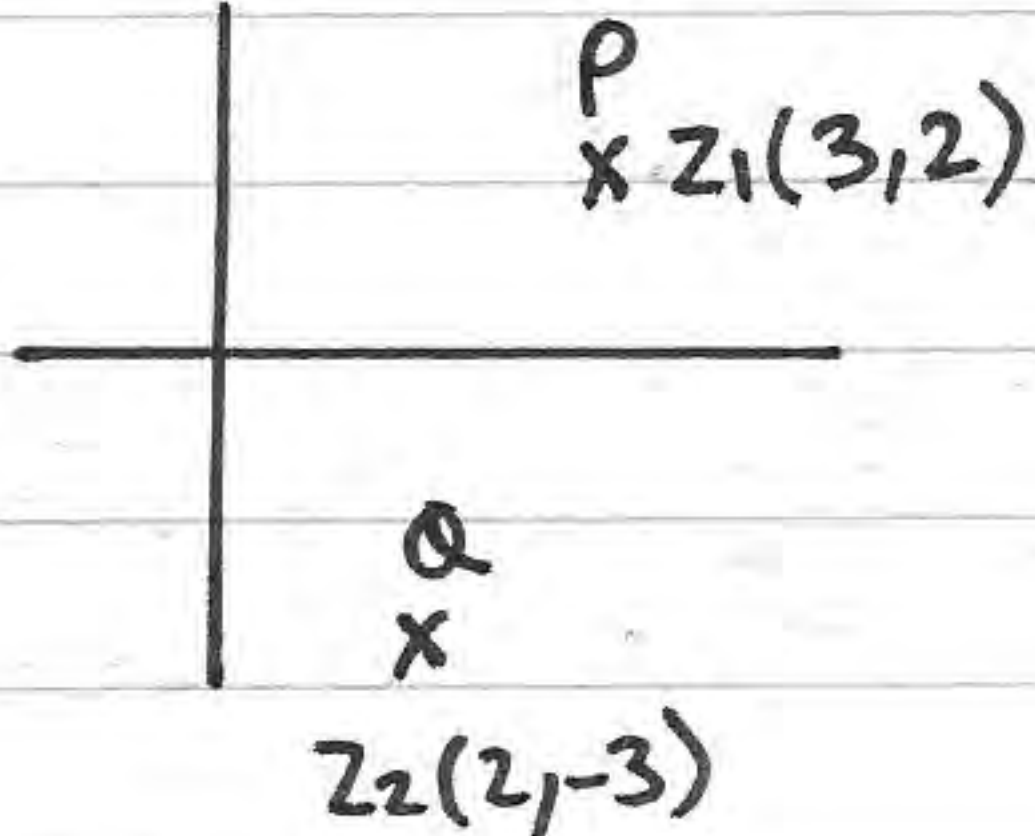
$$c) \quad \text{focus } (a, 0) \quad x=a \quad y = -qa + aq = 0 \neq$$

$$d) \quad \text{at directrix } x = -a \Rightarrow y = -q(-a) + aq = aq + aq \\ \Rightarrow y = 2aq \quad (-a, 2aq)$$

$$a) \quad Z_2 = \frac{12-5i}{3+2i} \times \frac{(3-2i)}{(3-2i)} = \frac{36-24i-15i+10i^2}{9-4i^2} = \frac{26-39i}{13}$$

$$Z_2 = 2-3i$$

b)



$P \times Z_1(3, 2)$

Q
 \times
 $Z_2(2, -3)$

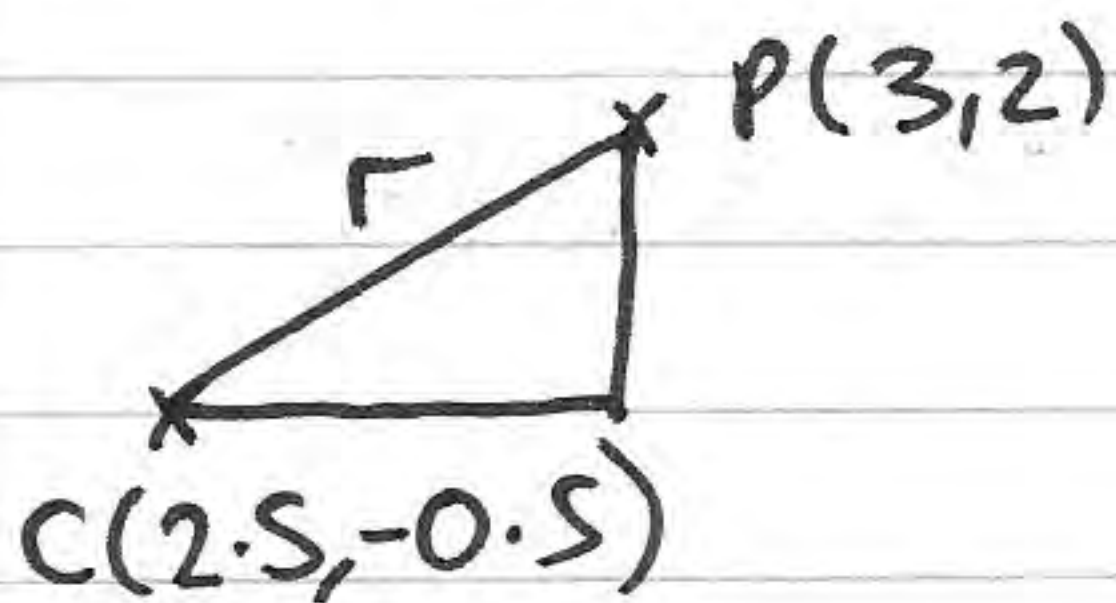
$$M_{OP} = \frac{2}{3} \quad M_{OQ} = -\frac{3}{2}$$

$$M_{OP} \times M_{OQ} = \frac{2}{3} \times -\frac{3}{2} = -1$$

hence perp $\Rightarrow \angle POQ = \frac{\pi}{2}$

c) $\angle POQ = \frac{\pi}{2} \Rightarrow PQ$ must be the diameter

$$\Rightarrow \text{midpoint } PQ = \text{centre} = \frac{5}{2} - \frac{1}{2}i$$



$$r^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2 \Rightarrow r = \sqrt{\frac{1}{4} + \frac{25}{4}}$$

$$\Rightarrow r = \frac{1}{2}\sqrt{26}$$

10) $A = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$ enlargement, centre origin,
Scale factor $3\sqrt{2}$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

reflection through $y=x$

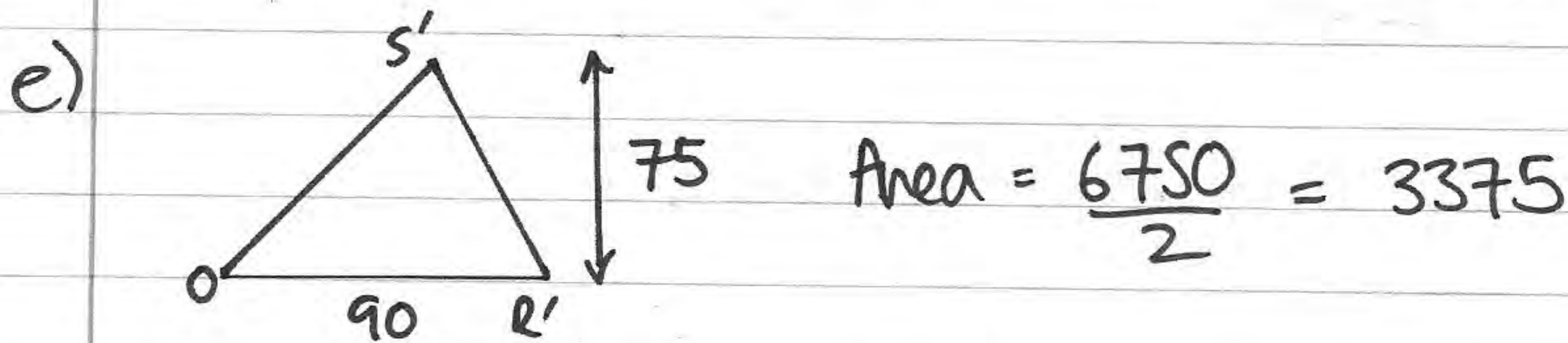
$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

rotation 45° , anticlockwise,
about origin.

$$b) D = CA = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$$

$$c) E = DB = \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$$

$$d) \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix} \quad \begin{array}{l} R'(90, 0) \\ S'(51, 75) \end{array}$$



$$\det E = -9 - 9 = -18 \quad \Rightarrow \text{Area ORS} = \underline{187.5}$$

$\div 18$