

D2 June 2010

1. (a) and (b)

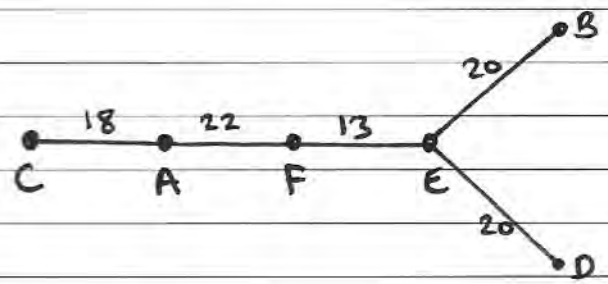
	✓ A	✓ B	✓ C	✓ D	✓ E	✓ F
✓ A	-	36	18	28	24	22
✓ B	36	-	54	22	20	27
✓ C	18	54	-	42	27	24
D	28	22	42	-	20	30
✓ E	24	20	27	20	-	13
✓ F	22	27	24	30	13	-

Prim's

- AC (18)
- AF (22)
- FE (13)
- EB (20)
- ED (20)

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93



b) Initial upper bound =  $93 \times 2 = \underline{186}$

(c)

	A	B	C	D	E	F
A	-	36	18	28	24	22
B	36	-	54	22	20	27
C	18	54	-	42	27	24
D	28	22	42	-	20	30
E	24	20	27	20	-	13
F	22	27	24	30	13	-

$A_{18} C_{24} F_{13} E_{20} B_{22} D_{28} A \rightarrow 125$

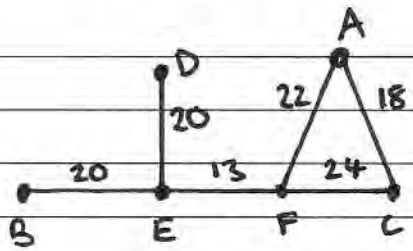
$A_{18} C_{24} F_{13} E_{20} D_{22} B_{36} A \rightarrow 133$

(d) Best upper bound: 125 ACFEBDA  
 Smaller, closer to optimal solution

(e)

	A	B	C	D	E	F
A	-	36	18	28	24	22
B	36	-	54	22	20	27
C	18	54	-	42	27	24
D	28	22	42	-	20	30
E	24	20	27	20	-	13
F	22	27	24	30	13	-

Prim's BA(20)  
 EF(13)  
 ED(20)  
CF(24)



RMST 77

+ AC(18) + AF(22)  $\Rightarrow$  lower bound = 117

117 < optimal route  $\leq$  125

(Total 11 marks)

2. You may not need to use all of these tables

(a)

	1	2	3	4
H	18	24	22	17
J	20	25	19	-
L	25	24	27	22
S	19	26	23	14

Maximise

$\therefore$  Subtract every element from 27

	1	2	3	4
H	9	3	5	10
J	7	2	8	20
L	2	3	0	5
S	8	1	4	13

$J_4 = 2 \times$  biggest element so that it is not selected.

	1	2	3	4
H	6	0	2	7
J	5	0	6	18
L	2	3	0	5
S	7	0	3	12

Reduce Rows

-3  
-2  
X  
-1

	1	2	3	4
H	4	0	2	2
J	3	0	6	13
L	0	3	0	0
S	5	0	3	7

Reduce Columns

Smallest uncovered = 2

-2 X X -5

2 lines  $\Rightarrow$  not optimal

	1	2	3	4
H	2	0	0	0
J	1	0	4	11
L	0	5	0	0
S	3	0	1	5

3 lines  $\Rightarrow$  not optimal

Smallest uncovered = 1

	1	2	3	4
H	2	1	0	0
J	0	0	3	10
L	0	6	0	0
S	2	0	0	4

4 lines required

$\therefore$  optimal

	1	2	3	4
H	<del>2</del>	<del>1</del>	0	<del>0</del>
J	0	<del>0</del>	<del>3</del>	<del>10</del>
L	<del>0</del>	<del>6</del>	0	0
S	<del>2</del>	0	<del>0</del>	<del>4</del>

(b)

Worker	Task
Harry	3
Jess	1
Louis	4
Saul	2

(22)

(20)

(22)

(26)

£

90

3. (a)

	A	B	C	D	Supply
X	18	31	4		53
Y			18	29	47
Demand	18	31	22	29	

Initial Solution

(b) You may not need to use all of these tables

		28	20	19	22
0	X	(28)	(20)	(19)	16
-5	Y	15	12	(14)	(17)

Shadow Costs

	A	B	C	D
X	X	X	X	-6
Y	-8	-3	X	X

Improvement Indices

	A	B	C	D
X	18 - $\theta$	31	4 + $\theta$	
Y	+ $\theta$		18 - $\theta$	29

entering cell = YA  
 $\theta = 18$

exiting cell = XA  
put 0 in YC so it is not degenerate

	A	B	C	D
X		31	22	
Y	18		0	29

New Solution

		20	20	19	22
0	X	28	(20)	(19)	16
-5	Y	(15)	12	(14)	(17)

Shadow Costs

	A	B	C	D
X	8	X	X	-6
Y	X	-3	X	X

Improvement Indices

	A	B	C	D
X		31	$22 - \theta$	$\theta$
Y	18		$0 + \theta$	$29 - \theta$

entering cell = XD  
 $\theta = 22$

exiting cell = XC

	A	B	C	D
X		31		22
Y	18		22	7

New Solution

		14	20	13	16
	A	B	C	D	
X	28	(20)	19	(16)	
Y	(15)	12	(14)	(17)	

Shadow Costs

(c) Improvement Indices

$$XA = 14 ; XC = 6 ; YB = -9$$

$\therefore$  Not optimal as there is still a negative improvement index.

4. (a) You may not need to use all these rows

mini max

Stage	State	Action	Destination	Value
1	G	GT	T	17 *
	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	$\max(22, 17) = 22 *$
		DH	H	$\max(31, 21) = 31$
	E	EH	H	$\max(34, 21) = 34 *$
		EI	I	$\max(39, 29) = 39$
	F	FI	I	$\max(52, 29) = 52 *$
	3	A	AD	D
AE			E	$\max(38, 34) = 38 *$
B		BE	E	$\max(44, 34) = 44 *$
C		CE	E	$\max(36, 34) = 36 *$
		CF	F	$\max(35, 52) = 52$
4		S	SA	A
	SB		B	$\max(39, 44) = 44$
	SC		C	$\max(41, 36) = 41$

SAEHT £38000

S A E H T 130  
37 38 34 21

average = 32.5

£32500

5. (a) Value of initial flow:  $8+3+30 = 41$   
 (b) Capacity of  $C_1$ :  $8+15+10+11+25 = 69$   
 Capacity of  $C_2$ :  $24+8+10+18+4 = 64$   
 (c)

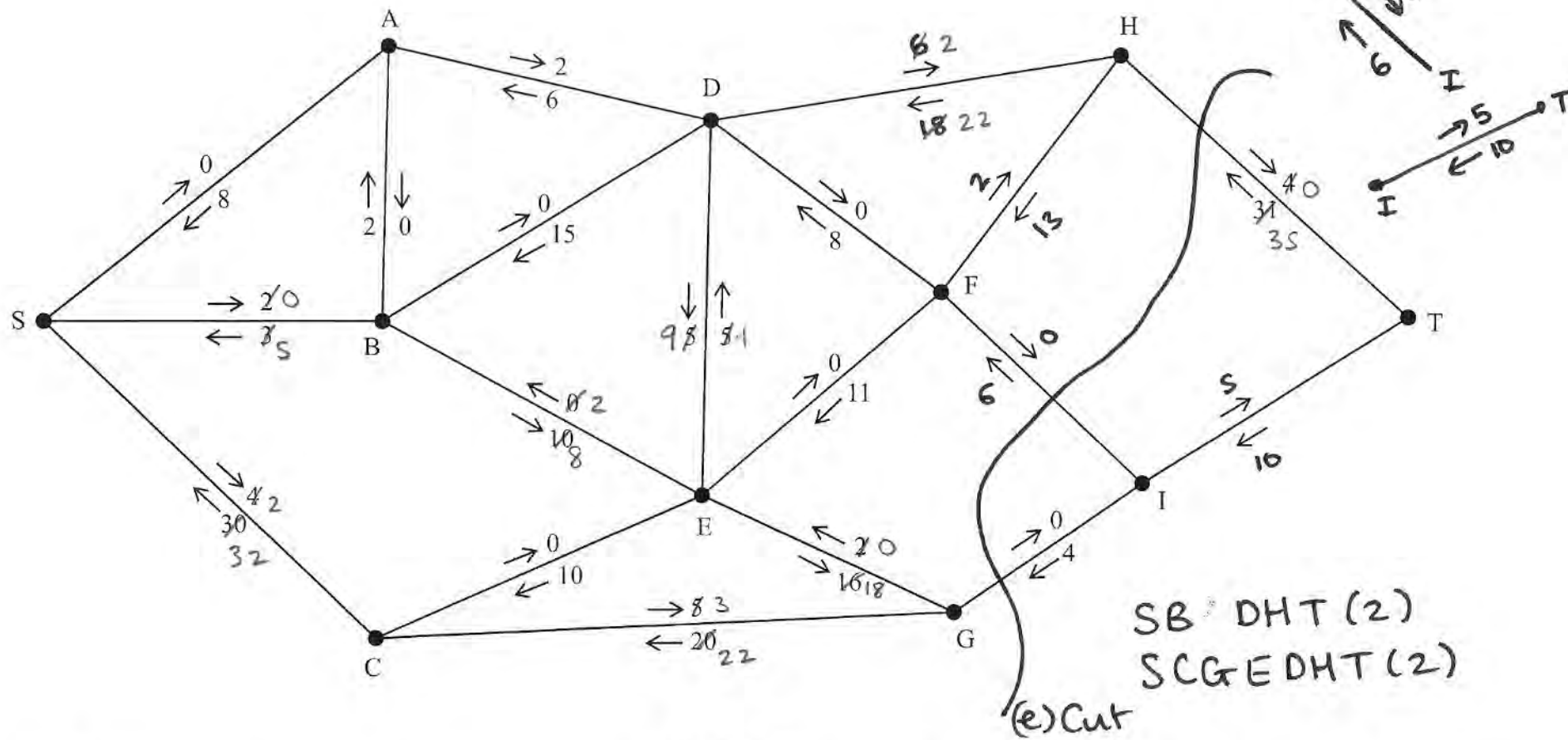


Diagram 1

Cut through HT, FI, GI Now only passes through saturated arcs.  
 $35+6+4 = 45 \therefore \text{min cut} = 45$  by min cut-max flow theorem  
 $\therefore \text{flow is maximal}$   
 max flow = 45



6. (a)

$$P - x - 2y - 6z = 0$$

b.v.	x	y	z	r	s	t	Value
r	0	1	2	1	0	0	24
* s	2	1	4	0	1	0	28
t	-1	$\frac{1}{2}$	3	0	0	1	22
P	-1	-2	-6	0	0	0	0

$$\theta = 24 \div 2 = 12$$

$$\theta = 28 \div 4 = 7^*$$

$$\theta = 22 \div 3 = 7\frac{1}{3}$$

You may not need to use all of these tableaux

(b)

Increase z

b.v.	x	y	z	r	s	t	Value	Row Ops
	0	1	2	1	0	0	24	X
*	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R2 \div 4$
	-1	$\frac{1}{2}$	3	0	0	1	22	X
P	-1	-2	-6	0	0	0	0	X

b.v.	x	y	z	r	s	t	Value	Row Ops
* r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$-2R2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	X
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	$-3R2$
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$+6R2$

$$\theta = 10 \div \frac{1}{2} = 20^*$$

$$\theta = 7 \div \frac{1}{4} = 28$$

$\theta$  - ve.

Question 6 continued

Increasing  $y$ 

b.v.	$x$	$y$	$z$	$r$	$s$	$t$	Value	Row Ops
	-2	1	0	2	-1	0	20	$R1 \div \frac{1}{2} (x2)$
	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	X
	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	X
$P$	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	X

b.v.	$x$	$y$	$z$	$r$	$s$	$t$	Value	Row Ops
$y$	-2	1	0	2	-1	0	20	X
$z$	0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$-\frac{1}{4}R1$
$t$	-3	0	0	$\frac{1}{2}$	1	1	6	$+\frac{1}{4}R1$
$P$	1	0	0	1	1	0	52	$+\frac{1}{2}R1$

$$\begin{aligned} x &= 0 \\ y &= 20 \\ z &= 2 \\ r &= 0 \\ s &= 0 \\ t &= 6 \\ P &= 52 \end{aligned}$$

	B plays 1	B plays 2	B plays 3
A plays 1	-4	5	1
A plays 2	3	-1	-2
A plays 3	-3	0	2

$V =$  value of original game to A

+5 to every element so that each is  $> 0$

	B1	B2	B3
A1	1	10	6
A2	8	4	3
A3	2	5	7

now let  $V = v + 5$  be the value of the new game to A.

A plays 1 prob =  $P_1$

A plays 2 prob =  $P_2$

A plays 3 prob =  $P_3$

$$P_1, P_2, P_3 \geq 0$$

If B plays 1  $V = P_1 + 8P_2 + 2P_3$

B plays 2  $V = 10P_1 + 4P_2 + 5P_3$

B plays 3  $V = 6P_1 + 3P_2 + 7P_3$

$\therefore$  objective is to maximise  $P = V$  such that  $P - V = 0$

Subject to

$$V \leq P_1 + 8P_2 + 2P_3$$

$$V \leq 10P_1 + 4P_2 + 5P_3$$

$$V \leq 6P_1 + 3P_2 + 7P_3$$

$$P_1 + P_2 + P_3 \leq 1$$

$$P_1, P_2, P_3 \geq 0$$