**Solutions**

| | | | | |
|---|---|---|---|---|
| **1.** | (a) | Any part of an optimal path is itself optimal | B1 | |
| | (b) | The route chosen such that the maximum arc length is as small as possible | B1 | |
| | (c) | e.g. Maximising freight by minimising fuel needed when planning multiple stage light aircraft journey | B2, 1, 0 | |

*B1 cao ("port", "section", OK; "arc", "stage", activity", "event", not)*
*B1 cao (not min of max rate, not minimize largest arc)*
*B2 cao*
*B1 cloze "Bod" gets B1*

**[4]**

**2.** Let $x_{ij}$ = 1 if worker does task, 0 otherwise — B1

where $x_{ij}$ indicates the arc from node i to node j i.e P, Q, R j E 1, 2, 3 — B1

| | | | | |
|---|---|---|---|---|
| $x_{p1} + x_{p2} + x_{p3} = 1$ | | $x_{p1} + x_{q1} + x_{r1} = 1$ | M1 | |
| $x_{q1} + x_{q2} + x_{q3} = 1$ | and | $x_{p2} + x_{q2} + x_{r2} = 1$ | A1 | |
| $x_{r1} + x_{r2} + x_{r3} = 1$ | | $x_{p3} + x_{q3} + x_{r3} = 1$ | A1 | 3 |

Minimise, $C = 8x_{p1} + 7x_{p2} + 3x_{p3} + 9x_{q1} + 5x_{q2} + 6x_{q3} + 10x_{r1} + 4x_{r2} + 4x_{r3}$
where C is in hundreds of pounds — B1, B1 — 2

*B1 cao*
*B1 defining variable – attempt*
*M1 at least 3* ***equations*** *– coefficients of one*
*A1 cao 3 correct*
*A1 cao 6 correct*
*B1 Minimise*
*B1 cao (condone a slip) (- accept cost in pounds)*

**[7]**

| | | | | |
|---|---|---|---|---|
| **3.** | (a) | Each activity must be visited once and then we return to the starting activity, this must be done in a minimum time | B2, 1, 0 | 2 |

*B2 cao – all 3 bits in the context*
*B1 cloze 'Bod' is B1 (e.g. not in contect; just 'each activity once' – but not all 3; …)*

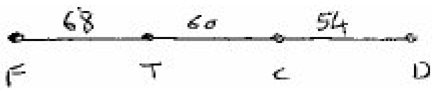
| | | | | |
|---|---|---|---|---|
| | (b) | 108 + 54 + 150 + 68 + 100 = 480 minutes (= 8 hours) | M1 A1 | 2 |

*M1 (maybe implicit) attempting to add 5 values*
*A1 cao*

(c) Use nearest neighbour B F T C D B  M1 A1

64 + 68 + 60 + 54 + 150 = 396 minutes (67 hours)  A1  3

*M1 each vertex visited once – either NN or 2 x mst-shortcut (BD)*
*A1 cao incl return to B (BFTCDB)*
*A1 cao (396)*

(d) 


CT, TF, CD (Prim or Kruskal)  M1 A1

182 +64 + 100 = 346 minutes  M1 A1ft  4

*M1 Finding correct minimum spanning tree (maybe implicit) 182 sufficient*
*A1 cao tree or 182*
*M1 adding 2 least arcs to B i.e. 100 and 64 only*
*A1ft cao ft from their m.s.t. value i.e. 164 and their* ***<u>tree</u>*** *length*

**[11]**

**4.** (a) Adding n $\geq 20$ to table to give  B1

| | H | P | R | W |
|---|---|---|---|---|
| A | 3 | 5 | 11 | 9 |
| B | 3 | 7 | 8 | N |
| C | 2 | 5 | 10 | 7 |
| D | 8 | 3 | 7 | 6 |

Reducing rows first $\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 4 & 5 & n-3 \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix}$ then columns $\begin{bmatrix} 0 & 2 & 4 & 3 \\ 0 & 4 & 1 & n-6 \\ 0 & 3 & 4 & 2 \\ 5 & 0 & 0 & 0 \end{bmatrix}$  M1 A13

Either $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$  M1 A1ft

↓ ↓

$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & n-7 \\ 0 & 1 & 2 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & n-8 \\ 0 & 1 & 3 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}$  M1 A1ft  4

A – H  P
B – R or R  cost £21 000
C – W  H  A1

| | | | | | |
|---|---|---|---|---|---|
| D – P W | | | | A1 | 2 |

(b) Not unique – gives the other solution M1 A1ft 2

**[11]**

**5.**

| Stage | State | Action | Value | | |
|---|---|---|---|---|---|
| 1 | H | HT | 4* | | |
| | I | IT | 3* | | |
| | J | JT | 12* | | |
| | K | KT | 20* | M1 A1 | 2 |
| 2 | D | DH | 2 +4 = 6 | | |
| | | DI | 4 + 3 = 7* | M1 A1 | |
| | E | EH | 3 + 4 = 7* | | |
| | | EI | 4 + 3 = 7* | | |
| | F | FJ | 10 + 12 = 22* | | |
| | | FK | –8 + 20 = 12 | | |
| | G | GJ | 10 + 12 = 22 | | |
| | | GK | 17 + 20 = 37* | A1 | 3 |
| 3 | A | AD | 3 + 7 = 10 | | |
| | | AE | 2 + 7 = 9 | M1 A1ft | |
| | | AF | –5 + 22 = 17* | | |
| | B | BD | 3 + 7 = 10 | | |
| | | BE | 2 + 7 = 9 | | |
| | | BF | –6 + 22 = 16* | | |
| | C | CF | 8 + 22 = 30* | | |
| | | CG | –15 + 37 = 22 | A1 ft | 3 |
| 4 | S | SA | 2 + 17 = 19 | | |
| | | SB | 3 + 16 = 19 | M1 A1ft | 2 |
| | | SC | –10 + 30 = 20* | | |

Route S C F J T £20 000 M1 A1 2

**[12]**

**6.** (a) Either e.g.

In an n × m problem, a degenerate solution occurs when the number of cells used is less than (n + m – 1) B2,1,0 2

or e.g. when all the demand for one destination is satisfied by all the supply from a source, before the final demand and supplies are allocated

*B2 cao*
*B1 cloze "bod" is B1*

| | | | |
|---|---|---|---|
| (b) | If the total supply > total demand a dummy is used to absorb the excess | B1 | 1 |
| | *B1 cao must (cannot decipher copy properly)* | | |

(c) $\begin{bmatrix} 15 & & \\ 1 & 11 & 0 \\ & & 17 \end{bmatrix}$ B1 1

*B1 cao total of five numbers*

(d) Shadow costs $S_A = 0$ $S_B = -1$ $S_C = -1$

$D_1 = 62$ $D_2 = 49$ $D_3 = 1$

Improvement indices $I_{A2} = 47 - 0 - 49 = -2*$

$I_{A3} = 0 - 0 - 1 = -1$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 49 = 10$

| | 1 (62) | 2 (49) | 3 (1) |
|---|---|---|---|
| (0) A | 15–θ | θ | |
| (–1) B | 1+θ | 11–θ | 0 |
| (–1) C | | | 17 |

M1A1A1ft 3

*Entering A2, exiting B2,* $\theta = 0$

Shadow costs $S_A = 0$ $S_B = -1$ $S_C = -1$

$D_1 = 62$ $D_2 = 47$ $D_3 = 1$

Improvement indices $I_{A3} = 0 - 0 - 1 = -1*$

$I_{B2} = 48 + 1 - 47 = 2$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 47 = 12$

| | 1 (62) | 2 (47) | 3 (1) |
|---|---|---|---|
| (0) A | 4–θ | 11 | θ |
| (–1) B | 12+θ | | 0–θ |
| (–1) C | | | 17 |

M1A1A1ft 3

*Entering A3, exiting B3,* $\theta = 0$

| | 1 (62) | 2 (47) | 3 (1) |
|---|---|---|---|
| (0) A | 4 | 11 | θ |
| (–1) B | 12 | | |
| (–1) C | | | 17 |

Shadow costs $S_A = 0$ $S_B = -1$ $S_C = 0$

$D_1 = 62$ $D_2 = 47$ $D_3 = 0$ M1 A1

Improvement indices $I_{B2} = 48 + 1 – 47 = 2$

$I_{B3} = 0 + 1 – 0 = 1$

$I_{C1} = 68 – 0 – 62 = 6$ B1

$I_{C2} = 58 – 0 – 47 = 11$

∴ Optimal

Cost 1497 units B1 4

**[14]**

**7.** (a) e.g. Maximise P = V B1

Subject to: $V – 5p_1 – 3p_2 – 6p_3 + r = 0$ M1

$V – 7p_1 – 8p_2 – 4p_3 + s = 0$ A2,1,0

$V – 2p_1 – 4p_2 – 9p_3 + t = 0$

$p_1 + p_2 + p_3 (+ u) = 1$

where V = value of game to A, $P_i$ = probability of A playing row i
$P_i \geq 0$ and r, s, t, u are stack variables all $\geq 0$ B1 5

*B1 Maximise/minimise and consistent function*
*M1 constraints (condone non-negativity)*
*– at least one correct <u>must be equations</u>*
*A2 all correct*
*A1 at least two correct*
*B1 defining variables*

(b) Not reducible and a three variable problem B1 1

*B1 cao – both*

(c) e.g.

| b v | V | $P_1$ | $P_2$ | $P_3$ | r | s | t | u | value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| r | **1** | –5 | –3 | –6 | 1 | 0 | 0 | 0 | 0 | M1 | |
| s | 1 | –7 | –8 | –4 | 0 | 1 | 0 | 0 | 0 | A1 | |
| t | 1 | –2 | –4 | –9 | 0 | 0 | 1 | 0 | 0 | | 2 |
| u | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | | |
| P | –1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | |

| b v | V | $P_1$ | $P_2$ | $P_3$ | r | s | t | u | value | Row ops | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| V | 1 | –5 | –3 | –6 | 1 | 0 | 0 | 0 | 0 | $R_1 / 1$ | M1 A1 | |
| s | 0 | –2 | –5 | **2** | –1 | 1 | 0 | 0 | 0 | $R_2 – R_1$ | A1 | |
| t | 0 | –3 | –1 | –3 | –1 | 0 | 1 | 0 | 0 | $R_3 – R_1$ | B1ft | |
| u | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | $R_4$ stet | | 4 |
| P | 0 | –5 | –3 | –6 | 1 | 0 | 0 | 0 | 0 | $R_5 + R_1$ | | |

| b v | V | $P_1$ | $P_2$ | $P_3$ | r | s | t | u | value | Row ops | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V | 1 | –11 | –18 | 0 | –2 | 3 | 0 | 0 | 0 | $R_1 + 6R_2$ | M1 A1ft |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_3$ | 0 | –1 | $-\frac{5}{2}$ | 1 | ½ | ½ | 0 | 0 | 0 | $R_2 / 2$ | A1 |
| t | 0 | 0 | $-\frac{17}{2}$ | 0 | $-\frac{5}{2}$ | $\frac{5}{2}$ | 1 | 0 | 0 | $R_3 + 3R_2$ | B1ft |
| u | 0 | 2 | $\frac{7}{2}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 1 | 1 | $R_4 - R_2$ | 4 |
| P | 0 | –11 | –18 | 0 | –2 | 3 | 0 | 0 | 0 | $R_5 + 6R_2$ | |

**[16]**

**8.** (a) $7x + 10y + 10z + r = 3600$

$6x + 9y + 12z + s = 3600$ B2,1,0

$2x + 3y + 4z + t = 2400$

$P - 35x - 55y - 60z = 0$ B2,0 4

(b) (i)

| b.v. | x | y | z | r | s | t | value | Row ops | | |
|---|---|---|---|---|---|---|---|---|---|---|
| r | 2 | $\mathbf{\frac{5}{2}}$ | 0 | 1 | $-\frac{5}{6}$ | 0 | 600 | $R_1 - 10R_2$ | A1 | |
| z | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 | 0 | $\frac{1}{12}$ | 0 | 300 | $R_2 \div 12$ | M1 | |
| t | 0 | 0 | 0 | 0 | $-\frac{1}{3}$ | 1 | 1200 | $R_3 - 4R_2$ | A1ft | |
| P | –5 | –10 | 0 | 0 | 5 | 0 | 1800 | $R_4 + 60R_2$ | B1 | 5 |

(ii)

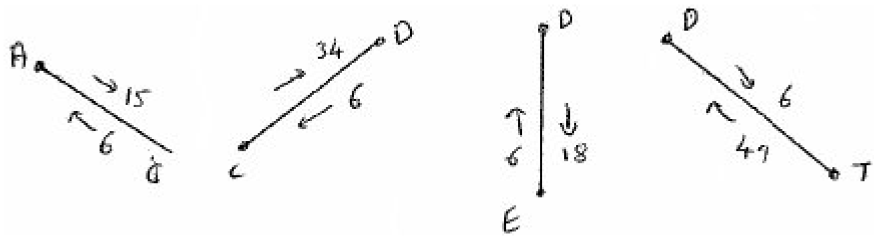
| b.v. | x | y | z | r | s | t | value | Row ops | M1 | |
|---|---|---|---|---|---|---|---|---|---|---|
| y | $\frac{4}{5}$ | 1 | 0 | $\frac{2}{5}$ | $-\frac{1}{3}$ | 0 | 240 | $R_1 \div \frac{5}{2}$ | M1 | |
| z | $-\frac{1}{10}$ | 0 | 1 | $-\frac{3}{10}$ | $\frac{1}{3}$ | 0 | 120 | $R_2 - \frac{3}{4} R_1$ | A1ft | |
| t | 0 | 0 | 0 | 0 | $-\frac{1}{3}$ | 1 | 1200 | $R_3$ stet | M1 | |
| P | 3 | 0 | 0 | 4 | $\frac{5}{3}$ | 0 | 20400 | $R_4 + 10R_1$ | A1 | 4 |

(c) P = 20400 x = 0 y = 240 z = 120 M1

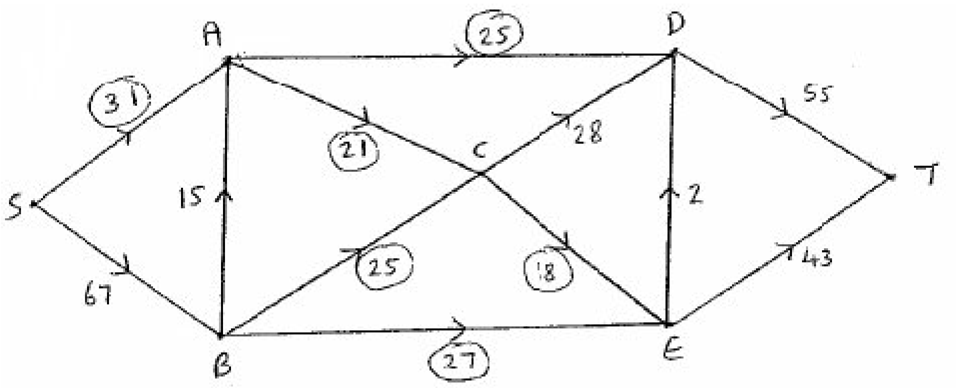
r = 0 s = 0 t = 1200 A2ft, A1ft, 0 2

**[16]**

| | | | | |
|---|---|---|---|---|
| **9.** | (a) | $C_1 = 103$, $C_2 = 177$, flow = 76 | B1, B1, B1 | 3 |
| | (b) | | M1A1 | 2 |




| | | | | |
|---|---|---|---|---|
| | (c) | e.g. S B C D T – 6 | M1 | |
| | | S B C D E T – 1 | A3,2,1,0 | |
| | | S B A C D E T – 15 | | |
| | | Max flow is 98 | B1 | 5 |
| | (d) | | M1A1 | 2 |




| | | | | |
|---|---|---|---|---|
| | (e) | Maximum flow = minimum cut | M1 | |
| | | Cut through AD, AC, BC and BE | A1 | 2 |

**[14]**