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## Core 3 - June 05 Solutions

1 a)  $\sin^2\theta + \cos^2\theta = 1 \quad \div \cos^2\theta$

$$\tan^2\theta + 1 = \sec^2\theta \quad \#$$

b)  $2\tan^2\theta + \sec\theta = 1$

$$2(\sec^2\theta - 1) + \sec\theta = 1$$

$$2\sec^2\theta + \sec\theta - 3 = 0$$

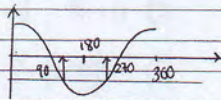
$$(2\sec\theta + 3)(\sec\theta - 1) = 0$$

$$\sec\theta = -\frac{3}{2} \quad \sec\theta = 1$$

$$\cos\theta = -\frac{2}{3} \quad \cos\theta = 1$$

$$\theta = 131.8^\circ \quad \theta = 0^\circ$$

$$\theta = 0^\circ, 131.8^\circ, 228.2^\circ, 360^\circ$$



$$2a) \text{ i) } \frac{d}{dx}(3\sin^2x + \sec 2x) = \frac{d}{dx}(3\sin^2x + (\cos 2x)^{-1})$$

$$= 6\sin x \cos x - (\cos 2x)^{-2} \cdot 2\sin 2x$$

$$= 6\sin x \cos x + \frac{2\sin 2x}{\cos^2 2x}$$

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$$\text{ii) } \frac{d}{dx}((x + \ln(2x))^3) = 3(x + \ln(2x))^2 \cdot \left(\frac{1+1}{2x}\right)$$

$$= 3(x + \ln(2x))^2 \cdot \left(\frac{1+1}{x}\right)$$

$$b) \quad y = \frac{5x^2 - 10x + 9}{(x-1)^2} \quad u = 5x^2 - 10x + 9 \quad u' = 10x - 10$$

$$v = (x-1)^2 \quad v' = 2(x-1)$$

$$\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2-10x+9)(2(x-1))}{(x-1)^4}$$

$$= \frac{(x-1)(10x-10) - 2(5x^2-10x+9)}{(x-1)^3}$$

$$= \frac{10x^2 - 10x - 10x + 10 - 10x^2 + 20x - 18}{(x-1)^3}$$

$$= \frac{-8}{(x-1)^3} \quad \#$$

3 a)  $\frac{5x+1}{x^2+x-2} - \frac{3}{x+2}$

$$= \frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$$

$$= \frac{5x+1 - 3(x-1)}{(x+2)(x-1)}$$

$$= \frac{2x+4}{(x+2)(x-1)} = \frac{2}{x-1} \quad \#$$

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b)  $y = \frac{2}{x-1}$

$$x-1 = \frac{2}{y}$$

$$x = \frac{2}{y} + 1 \Rightarrow F^{-1}(x) = \frac{2}{x} + 1$$

c)  $fg(x) = \frac{2}{x^2+4}$

$$fg(x) = \frac{1}{4} \quad \frac{2}{x^2+4} = \frac{1}{4}$$

$$x^2+4 = 8$$

$$x^2 = 4 \Rightarrow x = \pm 2 \quad (x=2 \text{ as } x > 1)$$

4. a)  $f(x) = 3e^x - \frac{1}{2}\ln x - 2$

$$f'(x) = 3e^x - \frac{1}{2x}$$

b) turning point  $f'(x) = 0$

$$3e^x - \frac{1}{2x} = 0$$

$$\frac{1}{x} = 6e^x \Rightarrow x = \frac{1}{6}e^{-x} \quad \#$$

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c)  $x_1 = \frac{1}{6}e^{-1} = 0.0613$

$$x_2 = \frac{1}{6}e^{-0.061313} = 0.1568$$

$$x_3 = \frac{1}{6}e^{-0.156754} = 0.1425$$

$$x_4 = \frac{1}{6}e^{-0.142485} = 0.1445$$

d)  $F'(0.14435) = 0.00206$

$$F'(0.14425) = -0.00069$$

change of sign indicates root:  $x = 0.1443$  (4dp) #

5. a)  $\cos 2A = \cos^2 A - \sin^2 A$

$$= 1 - \sin^2 A - \sin^2 A = 1 - 2\sin^2 A \quad \#$$

b)  $2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3$

$$= 2 \cdot 2\sin\theta\cos\theta - 3(\cos^2\theta - \sin^2\theta) - 3\sin\theta + 3$$

$$= 4\sin\theta\cos\theta - 3\cos^2\theta + 3\sin^2\theta - 3\sin\theta + 3$$

$$= 4\sin\theta\cos\theta - (3 - 3\sin^2\theta) + 3\sin^2\theta - 3\sin\theta + 3$$

$$= 4\sin\theta\cos\theta - 3 + 3\sin^2\theta + 3\sin^2\theta - 3\sin\theta + 3$$

$$= 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$$



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= sin(theta)(4cos(theta) + 6sin(theta) - 3)

c) 4cos(theta) + 6sin(theta) = Rsin(theta + alpha)
= Rsin(theta)cos(alpha) + Rcos(theta)sin(alpha)
Rcos(alpha) = 6 Rsin(alpha) = 4
tan(alpha) = 4/6 = 2/3 => alpha = 0.588
R^2 = 6^2 + 4^2 = 36 + 16
R = sqrt(52) = 2\*sqrt(13)
4cos(theta) + 6sin(theta) = 2\*sqrt(13)sin(theta + 0.588)

d) 2sin(2theta) = 3(cos(2theta) + sin(theta) - 1)
2sin(2theta) - 3cos(2theta) - 3sin(theta) + 3 = 0
sin(theta)(4cos(theta) + 6sin(theta) - 3) = 0 (part b))
sin(theta)(2\*sqrt(13)sin(theta + 0.588)) = 0
sin(theta) = 0 2\*sqrt(13)sin(theta + 0.588) - 3 = 0
theta = 0 sin(theta + 0.588) = 3/(2\*sqrt(13))

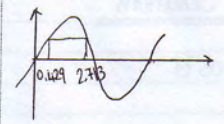
theta + 0.588 = 0.429 0.588 <= theta + 0.588 < pi + 0.588

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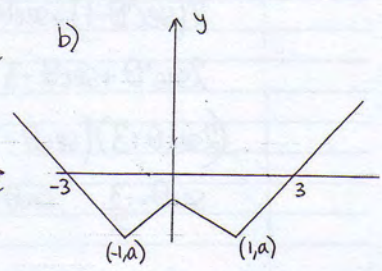
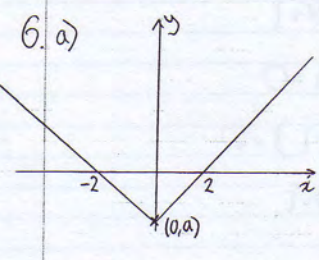
7. a) when t=0 p=300 300 = 2800ae^0 / (1 + ae^0)
300 = 2800a / (1 + a)
300 + 300a = 2800a
300 = 2500a
a = 300 / 2500 = 0.12 #

b) when p=1850 1850 = 2800 \* 0.12e^0.2t / (1 + 0.12e^0.2t)
1850 = 336e^0.2t / (1 + 0.12e^0.2t)
1850 + 222e^0.2t = 336e^0.2t
1850 = 114e^0.2t
0.2t = ln(1850/114)
t = 1/0.2 \* ln(1850/114) = 13.9
so 14 years

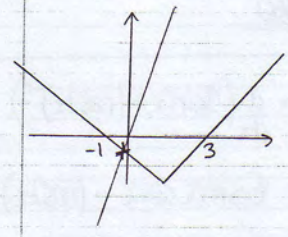
6



theta + 0.588 = 2.713
theta = 2.125
theta = 0, 2.125



c) a = -2, b = -1
d) |x-1| - 2 = 5x



one point of intersection
-(x-1) - 2 = 5x
-x + 1 - 2 = 5x
-1 = 6x
x = -1/6