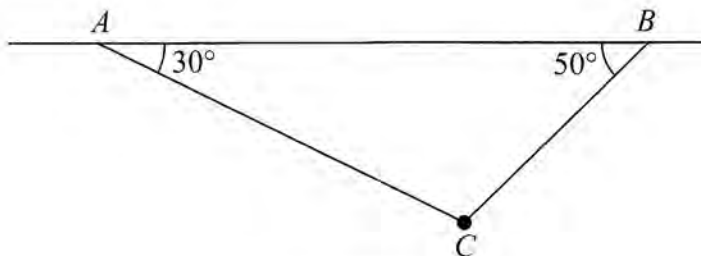1.

Figure 1

A particle of weight W newtons is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at $30°$ and $50°$ respectively, as shown in Figure 1.
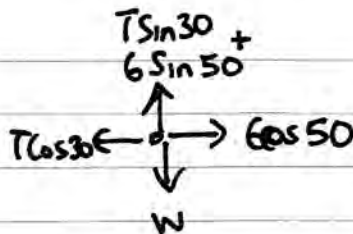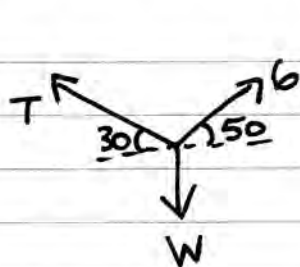
Given that the tension in BC is 6 N, find

(a) the tension in AC,

(3)

(b) the value of W.

(3)

a) $\therefore T\cos 30 = 6\cos 50$

$T = 4.45N$

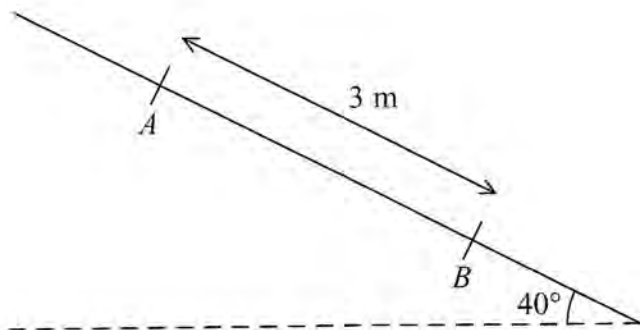b) $\therefore W = \frac{1}{2}T + 6\sin 50$

$W = 6.82N$

2.

Figure 2

A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B, as shown in Figure 2. A particle P of mass m kg is held at rest on the plane at A. The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

(a) Find the acceleration of P down the plane.

(5)

(b) Find the speed of P at B.

(2)

$NR = mg\cos 40 \quad \therefore fmax = \mu NR$
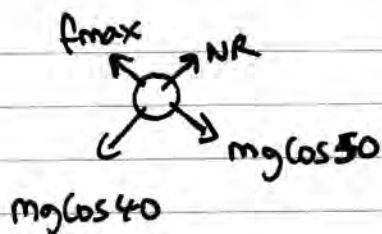
$fmax = \frac{1}{2}mg\cos 40$

$RF \leqslant ma \Rightarrow \cancel{m}g\cos 50 - \frac{1}{2}\cancel{m}g\cos 40 = \cancel{m}a$

$\therefore a = 2.5457\ldots$

$a = 2.55 \ (3sf)$

b) $S = 3$
   $u = 0$
   v
   $a = 2.5457\ldots$
   t

$v^2 = u^2 + 2as \qquad v^2 = 2(2.5457\ldots)3$

$\therefore v = 3.91 \ (3sf)$

3. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.

(a) Find the speed of the ball at the instant before it strikes the ground for the first time.

(2)

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time.

(2)

(c) Find the magnitude of the impulse on the ball in the first impact with the ground.

(2)

(d) Sketch, in the space provided, a velocity-time graph for the motion of the ball from the instant when it is released until the instant when it strikes the ground for the second time.
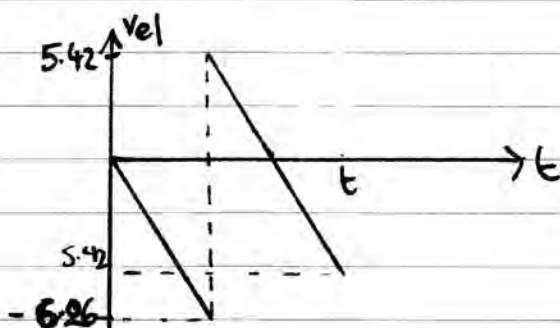
(3)

(e) Find the time between the instant when the ball is released and the instant when it strikes the ground for the second time.

(4)

a) $S = 2 \downarrow$ $v^2 = u^2 + 2as$ $\Rightarrow v^2 = 2(9 \cdot 8)2$
$u = 0$
v
$a = 9 \cdot 8 \downarrow$ $\therefore v = 2\sqrt{g} = 6.26099\ldots$ $\underline{6.26}$
t

b) $S = 1 \cdot 5 \uparrow$ $v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2(9 \cdot 8)(1 \cdot 5)$
u
$v = 0$
$a = -9 \cdot 8 \uparrow$ $\therefore u = \sqrt{3g} = \underline{5.42}$
t

c)   Initial mom $= 0.3 \times 2\sqrt{g} = 1.878\ldots$ $\therefore$ Impulse $= \underline{3.5}$
    final mom $= 0.3 \times -\sqrt{3g} = -1.62665\ldots$

d) let upwards $= +ve$

e) from part (a) Suvat $v = u + at$ $6.26 = 9.8t$ ∴ $t = 0.638877$
time taken to hit ground the first time.

ii) $s = 0$ $s = ut + \frac{1}{2}at^2$

   $u = \sqrt{3g}$ $0 = \sqrt{3g}\, t - 4.9t^2$
   ✓

   $a = -9.8$ $0 = t(\sqrt{3g} - 4.9t)$ $t = \frac{\sqrt{3g}}{\pm 9} = 1.106567$
   t
                 $t = 0$

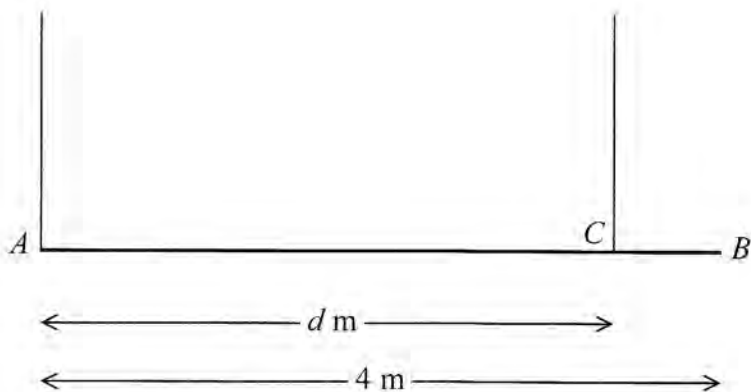∴ total time = 1.75 sec
                 2

4.

Figure 3

A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.
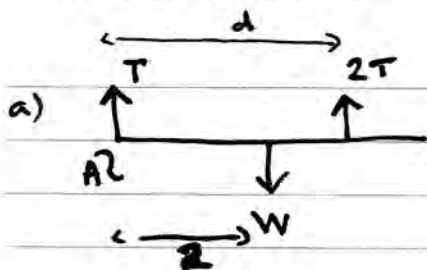
(a) Find the value of d.

(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.
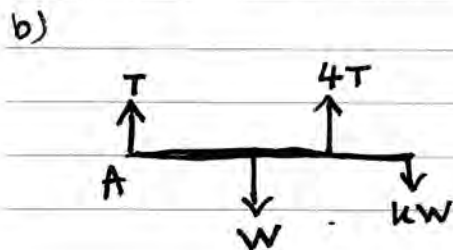
(b) Find the value of k.

(6)

a)

$A\circlearrowleft \quad 2W = 2T \times d \quad \therefore \quad d = \dfrac{W}{T}$

$\uparrow = \downarrow \quad 3T = W \quad \therefore \quad d = \dfrac{3T}{T} \quad d = \dfrac{3m}{2}$

b)

$\uparrow = \downarrow \Rightarrow 5T = (k+1)W$

$A\circlearrowleft \quad 2W + 4kW = 12T$

$2W + 4kW = \dfrac{12}{5}(k+1)W$

$2 + 4k = 2.4k + 2.4$

$1.6k = 0.4$

$\therefore k = \dfrac{0.25}{2}$

5. A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

(3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$ m s^{-1}.

(c) Find the distance moved by Q in 2 seconds.

(2)

(d) Show that at time $t = 3.5$ seconds both particles are moving in the same direction.

(3)

a) $F = ma \Rightarrow a = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \div \frac{1}{2} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ $|a| = \sqrt{6^2 + 4^2}$

$\therefore |a| = \sqrt{52} = 2\sqrt{13}$

b) $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \end{pmatrix}$ $13\mathbf{i} - 5\mathbf{j}$

c) $dist = |vel \times time| = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times 2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\therefore dist = \sqrt{4^2 + 2^2}$

$= 2\sqrt{5} = 4.47m$

d) $V_p = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 3.5\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 22 \\ -11 \end{pmatrix}$ $11V_q = \begin{pmatrix} 22 \\ -11 \end{pmatrix}$

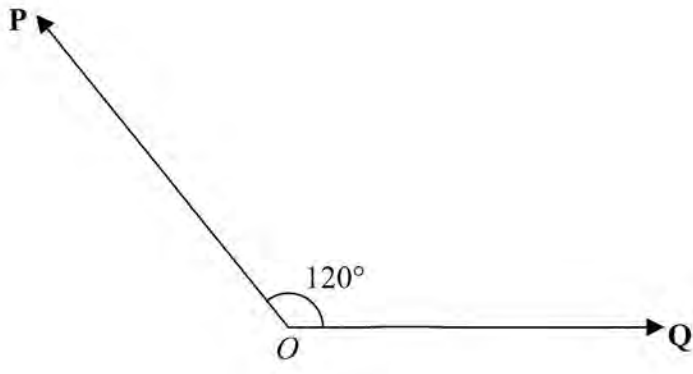$\therefore$ they are moving in the same direction when $t = 3.5$

6.

Figure 4

Two forces **P** and **Q** act on a particle at O. The angle between the lines of action of **P** and **Q** is $120°$ as shown in Figure 4. The force **P** has magnitude 20 N and the force **Q** has magnitude X newtons. The resultant of **P** and **Q** is the force **R**.

Given that the magnitude of **R** is $3X$ newtons, find, giving your answers to 3 significant figures

(a) the value of X,

(5)

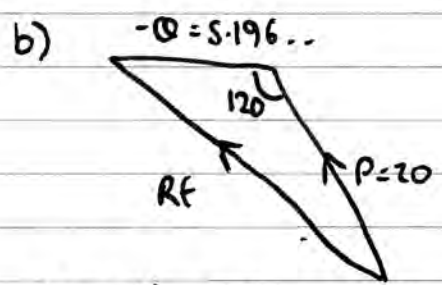(b) the magnitude of $(P - Q)$.

(4)
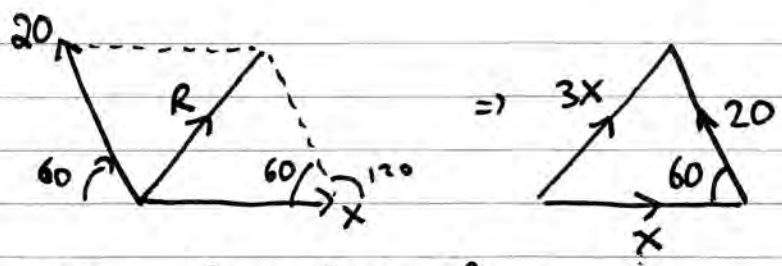
$$\cos 60 = \frac{x^2 + 20^2 - (3x)^2}{2(x)(20)} \Rightarrow 20x = x^2 + 400 - 9x^2$$

$$\therefore 8x^2 + 20x - 400 = 0$$

$$\Rightarrow 2x^2 + 5x - 100 = 0$$

$$\therefore x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 100}}{4}$$

$$x = 5.93$$

b) $-Q = 5.196..$

$|P-Q|^2 = 20^2 + (5.93)^2 - 2(20)(5.93...)\cos 120$
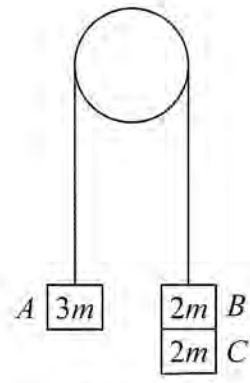
$$\therefore |P-Q| = 23.5$$

7.

Figure 5

Three particles A, B and C have masses $3m$, $2m$ and $2m$ respectively. Particle C is attached to particle B. Particles A and B are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5. The system is released from rest and A moves upwards.

(a) (i) Show that the acceleration of A is $\dfrac{g}{7}$

    (ii) Find the tension in the string as A ascends.

(7)

At the instant when A is 0.7 m above its original position, C separates from B and falls away. In the subsequent motion, A does not reach the pulley.
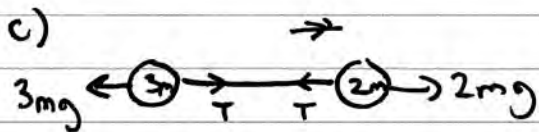
(b) Find the speed of A at the instant when it is 0.7 m above its original position.

(2)

(c) Find the acceleration of A at the instant after C separates from B.

(4)

(d) Find the greatest height reached by A above its original position.

(3)

a) $3mg \leftarrow$ ⬆a ⬅ $4mg$ $R\overleftrightarrow{F} = mg = 7ma$

$\qquad\qquad\qquad\qquad\qquad\qquad \therefore a = \frac{1}{7}g$
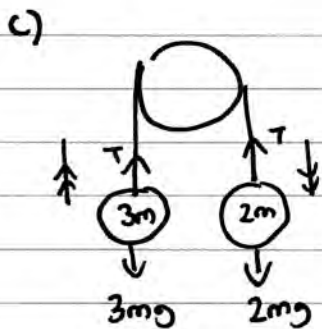
ii) $T - 3mg = 3ma \Rightarrow T = \frac{3}{7}mg + 3mg$

$\qquad\qquad\qquad \therefore T = \frac{24}{7}mg$

b) $S = 0.7$ $v^2 = u^2 + 2as$

$u = 0$

v $v^2 = 2(1.4)(0.7)$

$a = 1.4$ $v^2 = 1.96$

t $v = 1.4$

c)

$3mg \leftarrow \textcircled{3m} \xrightarrow{T} \quad \xleftarrow{T} \textcircled{2m} \rightarrow 2mg$

$3mg - 2mg$

c)

$2mg - 3mg = 5a$

$-mg = 5ma \quad \therefore a = -\frac{1}{5}g$

d)
$S =$
$u = 1.4$
$V = 0$
$a = -1.96$
t

$V^2 = u^2 + 2as$

$0 = 1.4^2 + 2(-1.96)s$

$S = \dfrac{1.4^2}{3.92} \quad \therefore S = 0.5m$

$\therefore gh = 1.2m$ above starting point